# Midterm 2

ⓘ This is a preview of the published version of the quiz

Started: Nov 15 at 3:16am

# Quiz Instructions

Midterm 2 is open book. You are allowed to use any lecture/course notes, homeworks, discussions, or websites (except those for collaborative documents or forums). In addition to this, we will allow the use of a calculator and a Python File or Notebook. You may not access or post on any collaborative documents (e.g. Google Docs) or forums (e.g. Chegg). **Collaboration with other students is prohibited.**

Assuming you do not have an approved time extension, you will have 1 hour (60 minutes) to complete the Midterm and you may begin the Midterm at any point during the window of 7:10-8:30 pm. However, the Midterm will close at 8:30 pm, meaning that you must start by 7:30 pm to have the full 1 hour. **We are not Zoom proctoring.**

We will not clarify anything during the exam so please do your best with the information provided. If you have an issue during your exam please email us at **eecs16b-fa20@berkeley.edu** (mailto:eecs16b-fa20@berkeley.edu) and CC the professors (**seth.sanders@berkeley.edu** (mailto:seth.sanders@berkeley.edu) and **mlustig@eecs.berkeley.edu** (mailto:mlustig@eecs.berkeley.edu) ).

Good luck!

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## Question 1 — 1 pts

Consider the following continuous-time system:

$$\frac{d}{dt}\vec{x}(t) = \begin{bmatrix} 1 & -1 \\ -6 & 0 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 1 \\ \alpha \end{bmatrix} u(t).$$

For which values of $\alpha$ is this system controllable?

**Mark all the correct options.**

0 [ Select ]

-1 [ Select ]

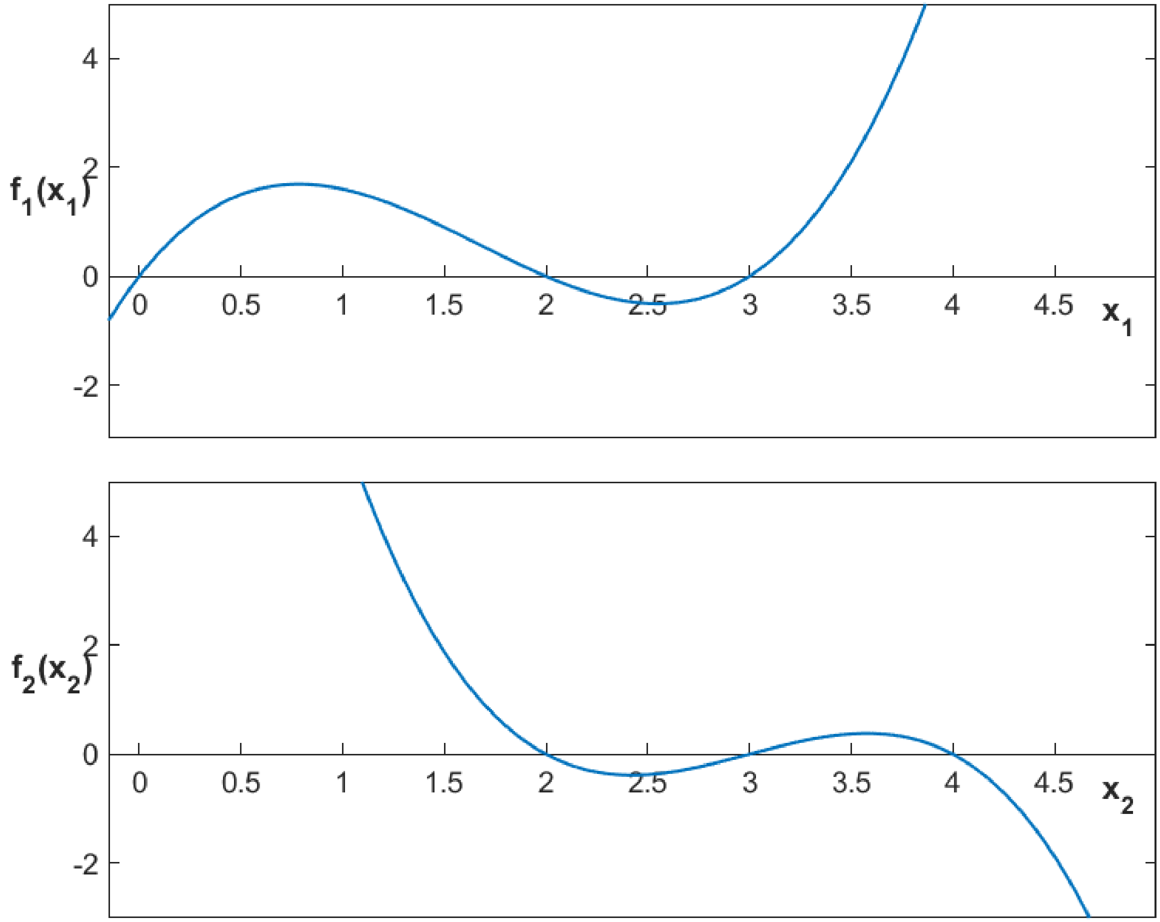
2 [ Select ]

3 [ Select ]

## Question 2

**2 pts**

Suppose we have a system $\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_1(x_1) \\ f_2(x_2) \end{bmatrix}$ with $f_1$ and $f_2$ plotted below.




How many equilibrium points are there with $0 \leq x_1 \leq 5$ and $0 \leq x_2 \leq 5$ ?

[ Select ]

For each of the following, state whether the system is stable or unstable when linearized about this point:

| | |
|---|---|
| $x_1 = 2, x_2 = 3$ | [ Select ] |
| $x_1 = 0, x_2 = 4$ | [ Select ] |

## Question 3

1 pts

Let us model a biological system as the following set of differential equations:

$$\frac{d}{dt}m = k_1 - d_1 m$$

$$\frac{d}{dt}p = k_2 m - d_2 p$$

Where $k_1, k_2, d_1, d_2$ are all constants, with units appropriate to the situation. If a biological system like this is allowed to persist for a long time, $m$ and $p$ will converge to a unique equilibrium. What are the values of $m$ and $p$ at this equilibrium?

$m =$ [ Select ]

(i) $\frac{k_1}{d_1}$ (ii) $-\frac{k_1}{d_2}$ (iii) 0 (iv) $\frac{-k_1 k_2}{d_1 d_2}$

$p =$ [ Select ]

(i) $\frac{k_1 k_2}{d_1 d_2}$ (ii) $-\frac{k_1 k_2}{d_1 d_2}$ (iii) 0 (iv) $\frac{k_2}{d_2}$

## Question 4

2 pts

We have the following discrete time system:

$$\vec{x}(t+1) = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(t),$$

$\vec{x}(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$.

Mark if the following statements about this system are **True/False**.

It is possible to design an input sequence $u(0), u(1), u(2), u(3)$ to reach any $\vec{x}^* \in \mathbb{R}^2$ at $t = 4$. [ Select ]

It is possible to design an input sequence $u(0), u(1)$ to reach $x(2) = \begin{bmatrix} 1+\alpha \\ -2-\alpha \end{bmatrix}$ for any scalar $\alpha \in \mathbb{R}$. [ Select ]

This system is controllable. [ Select ]

In a single time step, we can reach $x(1) = \begin{bmatrix} 1+\alpha \\ -2-\alpha \end{bmatrix}$ for any scalar $\alpha \in \mathbb{R}$. [ Select ]

## Question 5

**1 pts**

Consider the following discrete time linear system:

$$\vec{x}(t+1) = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t).$$

For feedback control $u(t) = \begin{bmatrix} k_1 & k_2 \end{bmatrix} \vec{x}(t)$, which of the following values of $k_2$ make the eigenvalues of the resulting closed loop system sum to zero?

- ○ -2
- ○ 2

○ 1

○ -1

## Question 6

**1 pts**

Taejin is trying to identify an unknown linear discrete-time system of the form $\begin{bmatrix} x(t+1) \\ y(t+1) \end{bmatrix} = \underbrace{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}}_{A} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + \underbrace{\begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix}}_{B} u(t)$

To do this, he applies the following sequence of scalar inputs

$u(0), u(1), \ldots, u(10), u(11)$

and observes the following states

$x(0), y(0), x(1), y(1), \ldots, x(11), y(11), x(12), y(12)$

Which of the following are valid set-ups that can be used to solve for the $A$ and $B$ matrices?

$$\begin{bmatrix} x(0) & y(0) & u(0) & u(0) \\ x(1) & y(1) & u(1) & u(1) \\ \vdots & \vdots & \vdots & \vdots \\ x(11) & y(11) & u(11) & u(11) \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ b_{11} & b_{21} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} x(1) & y(1) \\ x(2) & y(2) \\ \vdots & \vdots \\ x(12) & y(12) \end{bmatrix}$$

[ Select ]

$$\begin{bmatrix} x(0) & y(0) & u(0) \\ x(1) & y(1) & u(1) \\ \vdots & \vdots & \vdots \\ x(11) & y(11) & u(11) \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ b_{11} & b_{21} \end{bmatrix} = \begin{bmatrix} x(1) & y(1) \\ x(2) & y(2) \\ \vdots & \vdots \\ x(12) & y(12) \end{bmatrix}$$

[ Select ]

$$\begin{bmatrix} x(0) & y(0) & u(0) & 0 & 0 & 0 \\ 0 & 0 & 0 & x(0) & y(0) & u(0) \\ x(1) & y(1) & u(1) & 0 & 0 & 0 \\ 0 & 0 & 0 & x(1) & y(1) & u(1) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x(11) & y(11) & u(11) & 0 & 0 & 0 \\ 0 & 0 & 0 & x(11) & y(11) & u(11) \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ b_{11} \\ a_{21} \\ a_{22} \\ b_{21} \end{bmatrix} = \begin{bmatrix} x(1) \\ y(1) \\ x(2) \\ y(2) \\ \vdots \\ x(12) \\ y(12) \end{bmatrix}$$

[ Select ]

$$\begin{bmatrix} x(0) & y(0) & u(0) \\ x(1) & y(1) & u(1) \\ \vdots & \vdots & \vdots \\ x(11) & y(11) & u(11) \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ b_{11} & b_{21} \end{bmatrix} = \begin{bmatrix} x(0) & y(0) \\ x(1) & y(1) \\ \vdots & \vdots \\ x(12) & y(12) \end{bmatrix}$$

[ Select ]

## Question 7

**2 pts**

Given an over-constrained system of equations

$D\vec{p} = \vec{y}$

Which of the following statements must be true in order to create a **unique** Least-Squares estimate $\hat{\vec{p}}$.

$D$ has linearly independent columns. [ Select ]

$D^T D$ has linearly independent columns. [ Select ]

$D^T D$ has strictly positive eigenvalues. [ Select ]

The error $\vec{y} - D\hat{\vec{p}}$ is orthogonal to $\mathrm{Col}(D)$. [ Select ]

## Question 8

**1 pts**

Consider the following dynamical system with $a, d, \lambda, \mu > 0$,

$$\frac{d}{dt}x_1(t) = x_1(t)(a - bx_2(t) - \lambda x_1(t))$$
$$\frac{d}{dt}x_2(t) = x_2(t)(-d + cx_1(t) - \mu x_2(t))$$

Which of the following is the $A$ matrix when you linearize around the equilibrium of the form $(\frac{a}{\lambda}, 0)$.

- ○ $\begin{bmatrix} -a & -\frac{ab}{\lambda} \\ 0 & -d + \frac{ac}{\lambda} \end{bmatrix}$
- ○ $\begin{bmatrix} a + \frac{bd}{\mu} & -\frac{ab}{\lambda} \\ 0 & d \end{bmatrix}$
- ○

$$\begin{bmatrix} -a & 0 \\ -\frac{cd}{\mu} & -d + \frac{ac}{\lambda} \end{bmatrix}$$

○ $\begin{bmatrix} a + \frac{bd}{\mu} & 0 \\ -\frac{cd}{\mu} & d \end{bmatrix}$

## Question 9

**1 pts**

$A \in R^{2\times 2}$ is a diagonal matrix with singular values $\sigma_1 = 0.5, \ \sigma_2 = 0.1$. Complete the following statements:

When $A$ is the matrix in: $\vec{x}(t+1) = A\vec{x}(t) + Bu(t)$,

[ Select ] .

When $A$ is the matrix in $\frac{d}{dt}\vec{x}(t) = A\vec{x}(t) + Bu(t)$,

[ Select ] .

Quiz saved at 3:17am

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