EECS 16B
Designing Information Devices and Systems II

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Lecture 8: Bode Plots, Poles and Zeros

EECS 16B
Low-Pass Filter with RC Circuit

\[ V_{in}(t) \]
\[ R \]
\[ C \]
\[ + \]
\[ V_{out} \]
\[ - \]
Bode Plot – Single Pole

- **Magnitude**

- **Phase**

\[
H(j\omega) = \frac{1}{1 + j\frac{\omega}{\omega_c}}
\]
Low-Pass Filter with RC Circuit

\[ V_{out} = 0.86 \times 4 \, \text{V} = 3.44 \, \text{V} \]

is the peak value or 86% of the final value.

It reaches \(1 - e^{-\frac{t}{\tau}}\), where \(\tau = RC\).

What is the peak value of \(V_s(t)\)?

\[ \int_{0}^{\infty} R \, \frac{\mathrm{d}i}{\mathrm{d}t} \, L \, \mathrm{d}t = 0; \]

\[ \int_{0}^{\infty} \frac{v_{in}(t) - v_{out}(t)}{R} \, \mathrm{d}t = 0. \]

KVL around the loop:

\[ v_{in}(t) = v_{out}(t) + v_{c}(t); \]

\[ \int_{0}^{t} v_{in} \, \mathrm{d}t + \frac{1}{C} \int_{0}^{t} v_{out} \, \mathrm{d}t = \frac{1}{C} \int_{0}^{t} v_{c} \, \mathrm{d}t. \]

First Order Circuits: Forced Response

Example

Suppose a voltage pulse of width \(t_0\) at \(t = 0\). The output increases for \(\frac{t_0}{\tau}\) before changing the input again.

The input voltage pulse will decrease exponentially when \(V_{out}\) goes back down, \(V_{out}\) will increase exponentially toward 4 V.

When \(V_{in}\) switches between "low" (logic 0) and "high" (logic 1) voltage states, it will take a finite amount of time, referred to as "logic gates" are used to implement logical functions (NAND, NOR, NOT) in digital ICs.

Any logical function can be implemented using these gates.
Bode Plot – Pole and Origin Zero

\[ H(j\omega) = \frac{j\frac{\omega}{\omega_c}}{1 + j\frac{\omega}{\omega_c}} \]

- **Magnitude**

- **Phase**

\[ 20\log_{10}(|H_{HP}(j\omega)|) \text{[dB]} \]

\[ \angle H_{HP}(\omega) \]

- Shifted up by 90° compared with RC low pass filter.
Cascaded Networks

\[ V_{in} = V_{in1} \]
\[ V_{out} = V_{out1} + V_{out2} \]
\[ V_{out1} = V_{in2} \]
\[ V_{out2} = V_{out} \]

\[ H(f) = \frac{1}{1 + j\omega C_1} \times \frac{1}{1 + j\omega C_2} \]

\[ \omega_1 = \frac{1}{RC_1} \quad \omega_2 = \frac{1}{RC_2} \]

Assumed \( H_2(f) \) circuit does not "load down" \( H_1(f) \)
Cascaded Networks

- Needs to consider **source** and **load resistance**

\[
H(f) = H_1(f) \times H_2(f) = H_1(f)H_2(f)
\]

Unity Gain Amplifier

Need "Buffer"

- \(R_L = \infty\)
- \(R_S = 0\)

\(\infty\) Input Impedance

\(0\) Output Impedance
Cascaded Network with Unity Gain Buffer
Rational Transfer Functions

\[ H(j\omega) = K \frac{N(j\omega)}{D(j\omega)} \]

\[ H(j\omega) = K \frac{(j\omega)^{N_{z0}}}{(j\omega)^{N_{p0}} \left(1+j\frac{\omega}{\omega_{z1}}\right)\left(1+j\frac{\omega}{\omega_{z2}}\right)\ldots\left(1+j\frac{\omega}{\omega_{zn}}\right) \left(1+j\frac{\omega}{\omega_{p1}}\right)\left(1+j\frac{\omega}{\omega_{p2}}\right)\ldots\left(1+j\frac{\omega}{\omega_{pm}}\right)} \]

- \( N_{z0} \) zero frequencies
- \( N_{p0} \) pole frequencies
- \( \omega = 0 \) pole at origin
- \( \omega \neq 0 \) zeros at origin and poles at arbitrary points
Bode Plot – Single Pole

- \( H(j\omega) = \frac{1}{\left(1 + j\frac{\omega}{\omega_p}\right)} \)
Bode Plot – Single Zero

- \( H(j\omega) = \left(1 + j\frac{\omega}{\omega_{z1}}\right) \)
Bode Plot – Pole at Origin

- \( H(j\omega) = \frac{1}{j\omega} \)

\[
20 \log |H(j\omega)| = 20 \log |\frac{1}{j\omega}| = 20 \log |\frac{1}{j\omega}| = -20 \log \omega
\]
Bode Plot – Zero at Origin

- \( H(j\omega) = j\omega \)

\[ 20 \log |5\omega| = 20 \log (\omega) \]

\( \pm 20 \text{dB/dec} \)

90°
Rational Transfer Function – Example 1

\[ H(j\omega) = \frac{(1 + j\frac{\omega}{10^3})(1 + j\frac{\omega}{10^7})}{(1 + j\frac{\omega}{10^1})(1 + j\frac{\omega}{10^5})} \]

- **Hard way:**
  - List all pole and zero frequencies
    - \( 10, 10^3, 10^5, 10^7 \)
  - Asymptotic lines
    1. \( \omega \ll 10 \rightarrow H(j\omega) = 1 \)
    2. \( 10 < \omega \ll 10^3 \rightarrow \ldots \)

- **Easy Way:**
  1. Order all zeros and poles
  2. Initial points and slope at freq lower than all zero and pole
3) go from low to highest.
   (a) hit a “pole”
   Mag curve $\Rightarrow$ slope $-20$ dB/dec
   Phase $\Rightarrow$ Phase $-90^\circ$
   over $0 \leq \omega_c \leq 10 \omega_c$

(b) hit a “zero”
   Mag $\Rightarrow$ slope $+20$ dB/dec
   Phase $\Rightarrow$ $+90^\circ$

4) Whether zero, pole at origin $\Rightarrow$ define your initial point and slope
$10^1, 10^3, 10^5, 10^7$

$P = PZ$

$X \circ X \circ$

$w < 10$
General Rules for constructing rational transfer function

1. Order all poles and zero
2. Find initial magnitude & slope
   (a) pole/zero at origin
   \[
   \frac{(j\omega)^{N_{zo}}}{(j\omega)^{N_{po}}} = (j\omega)^{N_{zo} - N_{po}}
   \]

   \[\Rightarrow 0 \text{ dB at } \omega = 1\]
   \[\times \text{ Phase } (90^\circ) \times (N_{zo} - N_{po})\]

   \[\geq 20 \log |j\omega|^{N_{zo} - N_{po}} = 0\]

   \[\text{Slope } = (N_{zo} - N_{po}) \times 20 \text{ dB/dec} \]
(b) No pole/zero at origin

Starting point

Magnitude = \(20 \log |k|\)

Slope = 0

*Phase \(\hat{o} < K\)

\(20 \log k\)

\(\log \omega\)

At smaller than the smallest pole/zero

(3) From low \(\omega\) to high \(\omega\)

(a) see a pole

change slope by \(-20 \text{ dB/dec}\)

(b) see a zero

change slope by \(+20 \text{ dB/dec}\)
Mathematic Foundation

\[ 20 \log |H(j\omega)| = 20 \log |K| + \frac{N_{2o} - N_{po}}{20} \]

\[ + 20 \log \left| 1 + \frac{j\omega}{\omega_{p1}} \right| + \ldots + \]

\[ - 20 \log \left| 1 + \frac{j\omega}{\omega_{p1}} \right| - \ldots - \]
\[ H(j\omega) = \frac{(1 + j\frac{\omega}{10^3})(1 + j\frac{\omega}{10^7})}{(1 + j\frac{\omega}{10^5})(1 + j\frac{\omega}{10^1})} \]

![Graph of \( H(j\omega) \) showing phase and magnitude plots.]

-90° over 2 decades

+90° over 2 decades

-90° / 2 dec

+90° / 2 dec
Rational Transfer Function – Example 2

\[ H(j\omega) = \frac{(j\omega)(1 + j\frac{\omega}{10^3})(1 + j\frac{\omega}{10^7})}{(1 + j\frac{\omega}{10^1})(1 + j\frac{\omega}{10^5})} \]
Starting point is $j\omega$

$\omega = 1 \rightarrow 0$ dB, slope = $20$ dB/dec

$\angle(j\omega) = 90^\circ$
Rational Transfer Function – Example 3

\[ H(j\omega) = \frac{(1 + j \frac{\omega}{10^3})(1 + j \frac{\omega}{10^7})}{(j\omega)(1 + j \frac{\omega}{10^1})(1 + j \frac{\omega}{10^5})} \]

\[ \angle \frac{1}{j} = (0^\circ - 90^\circ) = -90^\circ \]
\[ 20 \log \left| \frac{1}{j\omega} \right| = -20 \log \omega \]

Rules:
- Pole: slope \(-20 \text{dB/dec}\)
- Zero: slope \(+20 \text{dB/dec}\)

Rules:
- Pole: phase \(\frac{\omega_p}{10} \rightarrow 10 \times \omega_p\) by \((-90^\circ)\)
- Zero: \(\frac{\omega_z}{10} \rightarrow 10 \times \omega_z\) by \((+90^\circ)\)
Those

$$\frac{1}{1 + j\frac{\omega}{\omega_B}}$$

$\text{dB}$

$\text{log } W$

$\text{Phase}$

$\frac{\omega_B}{\infty}$

$\text{log } \omega$
\[ H = \frac{R}{\frac{1}{j\omega C} + R} = \frac{j\omega RC}{1 + j\omega RC} = \frac{j\omega}{1 + j\frac{\omega}{\omega_B}}. \]

Single pole + zero @ 0 origin
\( z = a + jb \)

\( a, b \) both real numbers

\[ z = |z| \cdot \angle \phi \]

\[ |z| = \sqrt{a^2 + b^2} \quad \angle \phi = \tan^{-1} \left( \frac{b}{a} \right) \]

\( \angle \)

**Im**

**Re**

\( \phi \)

\( a \)

\( b \)

\( dB \) is defined for power

\( \text{power} \propto \text{voltage square} \)

\[ 10 \log |V|^2 = 20 \log |V| \]

\[ 50\% \text{ power} \to -3dB \]

**convention**