EECS 16B
Designing Information Devices and Systems II

Profs. Ming Wu, Shyam Parekh, Jean-Paul Tennant
Department of Electrical Engineering and Computer Sciences, UC Berkeley,
ingwu@berkeley.edu
Lecture 7: Transfer Functions

EECS 16B
Two-Port Network

• The RC circuit is an example of two-port network
Concept of Transfer Function

$H(j\omega) = \frac{V_{out}}{V_{in}}$

$H(j\omega)$ is a complex function of frequency
Transfer Function

- Transfer Function: \( H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)} \)
- Magnitude: \(|H(j\omega)|\)
- Phase: \(\angle H(j\omega)\)
Transfer Function

\( e^{j\omega t} \) \rightarrow \text{Circuit} \rightarrow H(j\omega) e^{j\omega t}

\[
\cos(\omega t) = \text{Re} \left[ e^{j\omega t} \right] \rightarrow \text{Circuit} \rightarrow \text{Re} \left[ H(j\omega) e^{j\omega t} \right]
\]
Low-Pass Filter with RC Circuit

\[ v_{in}(t) \]

\[ v_{out}(t) \]

\[ R \quad C \]

\[ + \quad V_{out} \quad \]

\[ - \]

\[ H(jw) = \frac{1}{1 + j \left( \frac{w}{\omega_B} \right)} \]

\[ \omega_B = \frac{1}{RC} \]

\[ H(f) = \frac{1}{1 + j \left( \frac{f}{f_B} \right)} \]
Frequency Response – Linear Plot
Frequency Response – Log-Log Plot

\[ |H(j\omega)| = \log 1 = 0 \]

\[ \omega \ll \omega_B, \quad H(j\omega) \rightarrow 1 \]

\[ \log |H(j\omega)| = \log \omega - \log \omega_B \]

\[ \frac{y}{d} = A - x \]

Intersect when \( \omega = \omega_B \)

\[ \omega = \omega_B \]

\[ \log |H(j\omega_B)| = \log \left| \frac{1}{j} \right| = \log \frac{1}{\sqrt{2}} \]
Decibels

- It is customary to express the magnitude of transfer function in dB scale

\[ |H(f)|_{db} = 20 \log_{10} |H(f)| \]

**Why 20?**

\[ 10 \log |V_{out}| = 20 \log |V_{in}| \]

**Table 6.2 Transfer-Function Magnitudes and Their Decibel Equivalents**

| \( |H(f)| \) | \( \log \) | \( |H(f)|_{db} \) |
|---|---|---|
| 100 | 2 | 40 |
| 10 | | 20 |
| 2 | 0.3 | 6 |
| \( \sqrt{2} \) | \( \frac{1}{2} \times 0.3 \times 20 \) | 3 dB |
| 1 | | 0 |
| \( 1/\sqrt{2} \) | | -3 dB |
| 1/2 | | -6 |
| 0.1 | | -20 |
| 0.01 | | -40 |

Copyright ©2016 Pearson Education, All Rights Reserved.
Logarithmic Frequency Scales

- **Decade**: 10x change in frequency
- **Octave**: 2x change in frequency
A **Bode** plot is a plot of the decibel magnitude of a network function vs log-scale frequency.
Bode Plot - Phase
Application of Low-Pass Filter

• Noise reduction

![Graph showing noise reduction](image_url)
High-Pass Filter with RC Circuit

\[ V_{in}(t) \]

\[ + \]

\[ V_{out} \]

\[ - \]
Low Pass Filter with RL Circuit
High Pass Filter with RL Circuit
Cascaded Networks
Series Resonance

\[ Z = R + j\omega L - \frac{j}{\omega C} \]
Series Resonance
Series Resonance
Series Resonance
Series Resonance
Series Resonance Bandpass Filter

Half-power frequencies are defined as the frequencies where the magnitude of the transfer function has fallen by a factor of \( \frac{1}{\sqrt{2}} = 0.707 \).

It can be shown that \( B = f_H - f_L = \frac{f_0}{Q_s} \).
Parallel Resonance
Parallel Resonance
Active Devices

![Diagram of an amplifier](https://example.com/diagram.png)

\[ v_i(t) = A_v v_i(t) \]

Copyright ©2018 Pearson Education, All Rights Reserved.