EECS 16B
Designing Information Devices and Systems II

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Lecture 3: Inductors and RL Circuits
Outline

- Inductance
- Inductors
- Differential Equations
- Mutual Inductors
- Transformers
Inductors

- Inductors store energy in the magnetic field
- Current carrying coils wound around a magnetic core material (popular materials are various types of iron oxides – often called ferrites) or “air core” inductors for higher frequencies

Symbol: 

Image source: Digikey
Physics of Inductors

- Inductance is the tendency of an electrical conductor to oppose a change in the electric current flowing through it
  - Current produces magnetic flux
  - Change of magnetic flux induces a voltage (Faraday’s Law)
  - For a coils with $N$ turns, area $A$, length $l$
    \[
    L = \frac{\mu N^2 A}{l}
    \]
    where $\mu$ is permeability

Unit of inductance: [H]: Henry
Magnetic vs Air Core Inductor

\[ L = \frac{\mu N^2 A}{l}, \text{ unit Henry, } [H] \]

\[ \mu = \mu_r \mu_0, \text{ unit: Henry per meter, } [H/m] \]

\[ \mu_0 = 4\pi \times 10^{-7} \text{ H/m: permeability of vacuum (air)} \]

\[ \mu_r: \text{ relative permeability} \]

- \( \mu_r = 1 \) for vacuum (exactly)
- \( \mu_r \approx 1 \) for air, most non-magnetic materials (Si, aluminum, copper, water, …)
- \( \mu_r \) large for magnetic materials, e.g., 5000 ~ 200,000 for iron, 20,000 ~ 50,000 for mu-metal (nickel iron alloy), 350 ~ 20,000 for Ferrite (manganese, zinc, nickel)
- \( \mu_r = 0 \) for superconductors

\[ \varepsilon = \varepsilon_r \varepsilon_0 \]
Inductors

Communication needs sending and receiving of \( C \) \( L \) Electro-Magnetic Wave
Current in an Inductor

\[ v(t) = L \frac{di}{dt} \]

\[ i(t) = \frac{1}{L} \int_{t_0}^{t} v \, dt + i(t_0) \]

Remember the capacitors

\[ i = C \frac{dv}{dt} \]

\[ v(t) = \frac{1}{C} \int_{t_0}^{t} i \, dt + v(t_0) \]
Stored Energy in an Inductor

\[ W = \frac{1}{2} CV^2 \quad \text{for cap} \]

\[ V = L \frac{di}{dt} \]

\[ [V] = [H] \frac{[A]}{[S]} \]

\[ [H] = \frac{[V]}{[A]} \cdot [S] \]

\[ = [S][S] \]

\[ p(t) = v(t)i(t) \]

\[ p(t) = i(t)L \frac{di(t)}{dt} = L \frac{i(t)di(t)}{dt} \]

\[ p(t)dt = Li(t)di(t) \]

\[ W = \int p(t)dt = \int L \frac{i(t)di(t)}{dt} = \frac{1}{2} L \int i^2 \]

\[ [S][S] \cdot [A]^2 = [W][S] \]

\[ = [S] \]

Power Consumed in Resistor R

\[ RC^2 \]

\[ \uparrow \]

\[ [S][S][A]^2 = [W][S] \]

\[ = [S] \]
Inductances in Series and Parallel

**Series: Common Current**

\[ V = L_1 \frac{dc}{dt} + L_2 \frac{dc}{dt} = (L_1 + L_2) \frac{dc}{dt} \]

\[ L_{eq} = L_1 + L_2 \]

*Same rule as resistor combo*

**Parallel: Common Voltage**

\[ V = V_1 = V_2 = L_1 \frac{du_1}{dt} = L_2 \frac{du_2}{dt} \]

\[ i = i_1 + i_2 \Rightarrow \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \]

\[ V = \frac{V_1}{L_1} + \frac{V_2}{L_2} \Rightarrow \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \]
Integrated Circuit Inductors

• Can’t build “3D” solenoid types so typically build spiral inductors. These are “tiny” (radius ~ thickness of hair)
Even if we try to avoid building an inductor, any closed loop circuit has intrinsic inductance!
Summary

Capacitors:

\[ i = C \frac{dv}{dt} \]
\[ w = \frac{1}{2} Cv^2 \]

- \( v \) cannot charge instantaneously
- \( i \) \textbf{can} charge instantaneously (do not short circuit a charged capacitor)

| \( N \) capacitors in series | \( \frac{1}{C_{eq}} = \sum_{i=1}^{N} \frac{1}{C_i} \) |
| \( N \) capacitors in parallel | \( C_{eq} = \sum_{i=1}^{N} C_i \) |

Inductors:

\[ v = L \frac{di}{dt} \]
\[ w = \frac{1}{2} Li^2 \]

- \( i \) cannot charge instantaneously
- \( v \) \textbf{can} charge instantaneously (do not open an inductor with current)

| \( N \) inductors in series | \( L_{eq} = \sum_{i=1}^{N} L_i \) |
| \( N \) inductors in parallel | \( \frac{1}{L_{eq}} = \sum_{i=1}^{N} \frac{1}{L_i} \) |
General Solution of the Differential Equation

For a first order, linear differential equation of the form:

\[
\frac{dy}{dt} + ay(t) = b(t) \quad \text{where we assume } a \text{ to be a constant}
\]

### Homogeneous Solution

\[
\frac{dy}{dt} + ay(t) = 0
\]

\[
\Rightarrow \frac{dy}{y} = -a dt
\]

\[
\Rightarrow \ln(y) = -at + C
\]

\[
\Rightarrow y(t) = Ke^{-at}
\]

### Particular Solution (Integrating Factor Method):

\[
\frac{dy}{dt} + ay(t) = b(t)
\]

We want to find a multiplier function \( f(t) \)

\[
f(t) \frac{dy}{dt} + af(t)y(t) = b(t)f(t)
\]

can be written as

\[
\frac{d}{dt} [y(t)f(t)] = b(t)f(t) \quad \text{(A)}
\]

For equation (A) to hold

\[
\frac{df(t)}{dt} = af(t)
\]

\[
\Rightarrow f(t) = e^{at}
\]

Then from (A)

\[
y(t) = \frac{1}{f(t)} \int b(t)f(t) dt
\]

\[
\Rightarrow y_p(t) = e^{-at} \int e^{at} b(t) dt
\]

\[
y(t) = Ke^{-at} + e^{-at} \int e^{at} b(t) dt
\]

\( K \) is determined using initial condition
R-L Circuits

For \( t > 0 \), KVL

\[
V_s = R\frac{di}{dt} + L\frac{d^2i}{dt^2}
\]

\[
\frac{di}{dt} + \frac{R}{L}i = \frac{1}{L}V_s
\]

\[
\frac{d^2i}{dt^2} + \frac{1}{L} \cdot \frac{R}{L}i = \frac{1}{L} \frac{V_s}{R}
\]

\[
i = \left(\frac{V_s}{R}\right) \left[ 1 - e^{-\frac{t}{\tau}} \right]
\]

\[
\frac{V_s}{R}
\]

\[
t
\]
\[ V_S = V_{\text{R}} + V \]
\[ = C \frac{dV}{dt} \cdot R + V \]
\[ \Rightarrow \frac{dV}{dt} + \frac{1}{RC} V = \frac{1}{RC} V_S \]

\[ V = V_0 \cdot (1 - e^{-\frac{t}{\tau}}) \]
\[ \tau = RC \]
Steady State

Capacitors:

\[ \frac{1}{T} v = C \frac{dv}{dt} = 0 \]

open circuit

Inductors:

\[ L \frac{di}{dt} = 0 \]

Short circuit
Same Equations $\Rightarrow$ Same Solution

$\frac{V_s}{R} \left[ 1 - e^{-\frac{t}{RC}} \right]$
Mutual Inductance

- Mutual inductance occurs when two windings are arranged so that they have a mutual flux linkage.
- The change in current in one winding causes a voltage drop to be induced in the other.

\[
V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}
\]
\[
V_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}
\]

Transformers (adapters), motors, generators (electric cars)
Flux Linkage

- Magnetic fields vary in time and space. Circuits that “cut” into flux will experience electromagnetic induction.
- Note: These are not intentional transformers!
The Dot Convention

- Dot indicate the polarity of coupling
- Total voltage induced in a coil is a summation of its own induced voltage and the mutually induced voltage

\[ v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \]

\[ v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \]
Transformers

- A common magnetic core is used to boost inductance
- By varying the turns ratio, we can boost the voltage or current
RFID : Transformer at a Distance!

- Card keys, contactless payment, inductive charging
Wireless Charging

1 Transmitter Coil
2 Receiver Coil
3 Current in Transmitter Coil
4 Electromagnetic Field
5 Current in Receiver Coil
6 Battery Charging
Why We Twist Wires!

- Twisted pair has a spatially varying magnetic flux that cancels (it flips orientation). Many such twisted pairs can be bundled together and used to send signals over long distances. This minimizes interference.