Lecture 3

- Computing: Transistors & Logic
  + RC transients [finish]
  + Non-homogeneous diff. eqns.
    + constant input
    + piece-wise constant input
    + continuous input

- Scaled

\[ V_x(t) = V_{dd} e^{-\frac{t}{\tau}}, \quad t \geq 0 \]

\[ V = R_{on,m} \cdot (C_{gm} + C_{gs}) \]

determines the speed of transition

\[ \tau_1 < \tau < \tau_2 \]

What happens for

If you make transistors smaller \( \Rightarrow B \leq V \)

Demand scaling \( \Rightarrow \) Moore's law (economics law)
KCL: \[ I_1 + I_2 + I_3 = 0 \]

Elements:
\[ V_1 = I_1 \cdot R_{on, pm} \]
\[ I_2 = C_{m2} \cdot \frac{dV_2}{dt} \]
\[ I_3 = C_{op2} \cdot \frac{dV_3}{dt} \]

\[ V_1 = V_x - Vdd \]
\[ V_2 = V_x \]
\[ V_3 = V_x - Vdd \]

From KCL & Elements:
\[ \frac{V_1}{R_{on, pm}} + C_{m2} \frac{dV_2}{dt} + C_{op2} \frac{dV_3}{dt} = 0 \]
\[ I_1 \quad I_2 \quad I_3 \]

From voltages:
\[ \frac{V_x - Vdd}{R_{on, pm}} + C_{m2} \frac{dV_x}{dt} + C_{op2} \frac{dV_x}{dt} (V_x - Vdd) = 0 \]
\[ \int \frac{dVdd}{dt} = 0 \]
\[ \frac{V_x - Vdd}{R_{on, pm}} + C_{m2} \frac{dV_x}{dt} + C_{op2} \frac{dV_x}{dt} = 0 \]

(1) \[ \frac{V_x - Vdd}{R_{on, pm}} + (C_{m2} + C_{op2}) \frac{dV_x}{dt} = 0 \]

\[ \frac{dV_x}{dt} = \frac{-V_x}{R_{on, pm} (C_{m2} + C_{op2})} + \frac{Vdd}{R_{on, pm} (C_{m2} + C_{op2})} \]

\[ \text{homogeneous term} \quad \text{non-homogeneous term} \]
Form: \[ \frac{d}{dt} x(t) = \lambda x(t) \] (homogeneous)
\[ \frac{d}{dt} x(t) = \lambda x(t) + a \] (non-homogeneous)

Go back to (1)
\[ \frac{V_x - Vdd}{R_{on,1n}} + (C_{on2} + C_{onp2}) \cdot \frac{d}{dt} (V_x - Vdd) = 0 \]

Try change of variables to transform the problem into one we know how to solve.
\[ \tilde{V}_x = V_x - Vdd \]
\[ \frac{\tilde{V}_x}{R_{on,1n}} + (C_{on2} + C_{onp2}) \cdot \frac{d}{dt} \tilde{V}_x = 0 \]

We already know how to solve this.
\[ \tilde{V}_x(t) = \tilde{V}_x(0) \cdot e^{-\frac{t}{\tilde{V}_x}} , \ t \geq 0 \]
\[ V_x(t) = Vdd - (V_x(0) - Vdd) \cdot e^{-\frac{t}{Vdd}} \]
\[ V_x(0) = 0V \]
Consider:

\[ V_{in}(t) \xrightarrow{\text{how short?}} V_{x}(t) \xrightarrow{\text{shrink}} \]

Will \( V_x(t) \) be able to follow these changes as a "logic" signal (i.e. to reach 0V or Vdd)?

\[ V_{dd} \quad V_{dd} \cdot \frac{t_1}{T_1} \quad V_{dd} \cdot \frac{t_2}{T_2} \quad V_{dd} \cdot \frac{t_3}{T_3} \quad \cdots \]

\[ V_{in}(t) \quad V_{in}(t) \quad V_{in}(t) \quad V_{in}(t) \quad \cdots \]

\[ t_0 \quad t_0 + T_1 \quad t_0 + T_1 + T_2 \quad t_0 + 2T_1 + 2T_2 \]

\[ V_{x}(t) \quad V_{x}(t) \quad V_{x}(t) \quad V_{x}(t) \quad \cdots \]

\[ \text{Will } V_{x}(t) \text{ be able to follow these changes as a "logic" signal (i.e. to reach 0V or Vdd)?} \]
\[ T_1 < T < T_2 \]  

Limit condition is no dryer. It is actually \( V_x(t_0 + T_1) = \)
\[ = Vdd : e^{\frac{t_0 + T_1 - t_0}{T}} = \]
\[ = Vdd : e^{\frac{T}{T_2}} \]

Use previous \( V_x(t) \) solution as an initial condition for the next interval.

Solutions for piece-wise constant input:

Form: \( \frac{d}{dt} x(t) = \lambda x(t) - \lambda u(t) \in \text{constant} \)

or piece-wise constant

What we also want to solve is \( u(t) = u_c(t) \)

\( u_c(t) \)

\[ u(t) \neq u_c(t) \]

\[ u(t) \to u_c(t) \]

\[ \Delta \to 0 \]

\( u(t) = \text{const}, t \in [t_0, t_0 + \Delta] \)

Iterate & use the previous solution as initial condition

Then let \( \Delta \to 0 \) for \( u(t) \) to approach \( u_c(t) \)
in the limit (disc. + hw)
Why do we want to know the response to continuous time input?

EECS 16AB Pipeline

- Sense analog inputs
- Process digital inputs
- Actuate
- Sense output

Sensing: brain signals or voice signals

Signal of interest + interference

Design sensor cnets to ride this up, unwanted

The goal:
Filter/select signals of interest and reject interference.

How can a circuit become a filter - process the input continuous-time signal?