Lecture 2

* Computing: Transistors & Logic
* Transistor RC model
* Solving RC circuits
* RC transients

MOSFET (metal-oxide semiconductor field effect transistor) invented in 1955-60 by Atolia & Kailing

NMOS = n-channel MOSFET
PMOS = p-channel MOSFET

In the past and some present:

Planar devices

NMOS

\[
\begin{align*}
\text{S} & \quad \text{G} \\
\text{P} & \quad \text{D} \\
\text{Silicon} \\
\end{align*}
\]

PMOS

\[
\begin{align*}
\text{S} & \quad \text{G} \\
\text{G}_{on} (\text{gate capacitance}) \\
\text{R}_{on} (\text{channel on-resistance}) \\
\text{V}_{gs} > \text{V}_{thn} \Rightarrow \text{ON} \\
\text{V}_{gs} < \text{V}_{thn} \Rightarrow \text{OFF} \\
\end{align*}
\]
Today: 

FinFET channel length = Snm

Future:

For more details take 105, 151, 150

To analyze this RC model we need to understand RC circuits.

Solving RC circuits:

Elements: 

\[ I_2 = C \cdot \frac{dV_x}{dt} \]

\[ V_x = I_1 \cdot R \]

KCL:

\[ I_1 + I_2 = 0 \]

Want to find \( V_x(t) \) for \( t > 0 \).

\[ I_1 = \frac{V_x}{R} \Rightarrow \frac{V_x}{R} + C \frac{dV_x}{dt} = 0 \]

\[ I_2 = C \frac{dV_x}{dt} \]

\[ \frac{dV_x}{dt} = -\frac{V_x}{RC} \]

First order differential equation

Guess: \( V_x(t) = a \cdot e^{bt} \) (educated guess based on the properties of the derivative)
Initial condition:

\[ t=0: \quad V_x(0) = a \cdot e^{b \cdot 0} = a \]

\[ \frac{d}{dt} V_x(t) = \frac{1}{R} \left( a \cdot e^{b \cdot t} \right) = a \cdot b \cdot e^{b \cdot t} = b \cdot V_x(t) \]

Since \[ \frac{dV_x}{dt} = -\frac{V_x}{RC} \Rightarrow b = -\frac{1}{RC} \]

\[ V_x(t) = V_x(0) e^{-\frac{t}{RC}} \]

is a solution to

\[ \frac{dV_x}{dt} = -\frac{V_x}{RC} \]

\[ \frac{dV_x}{dt} \bigg|_{t=0} = -\frac{V_x(0)}{RC} \]

\[ T \quad \text{is the time constant of the RC circuit} \]

Check for uniqueness of the guess:

Suppose \( y(t) \) which also solves D.E.

\[ x(t) = x_0 \quad (1) \quad V_x(t) = y(t) \]

Shorter notation:

\[ \frac{d}{dt} x(t) = x(t) \quad (2) \quad b = -\frac{1}{RC} \]
In (1) we guessed & checked that

\[ x_d(t) = x_0 \cdot e^{\lambda t}, \quad t \geq 0 \]

i.e. satisfies (1) + (2)

In (2) need to prove \( y(t) = x_d(t) \)

i.e. the solution is unique.

Either prove \( \frac{dy(t)}{dx_d(t)} = 1 \) \( \Rightarrow y(t) - x_d(t) = 0 \)

\[ \frac{y(t)}{x_d(t)} = \frac{y(t)}{x_0 e^{\lambda t}} \]

For \( y(t) \) is a solution:

\[ \int_{x_0}^{y(t)} \frac{1}{y(t)} dy(t) = \int_{x_0}^{x_d(t)} \frac{1}{x_0 e^{\lambda t}} dx_0 \]

\[ \frac{d}{dt} \left( \frac{y(t)}{x_d(t)} \right) = \frac{d}{dt} \left( \frac{y(t)}{x_0 e^{\lambda t}} \right) = \frac{1}{x_0} \frac{d}{dt} \left( y(t) \cdot e^{-\lambda t} \right) = \]

\[ = \frac{1}{x_0} \left( \frac{d}{dt} y(t) \cdot e^{\lambda t} + y(t) (-\lambda) \cdot e^{-\lambda t} \right) = \]

\[ = \frac{1}{x_0} \left( \lambda y(t) e^{-\lambda t} - \lambda y(t) e^{-\lambda t} \right) = 0 \]

\( \Rightarrow \frac{y(t)}{x_d(t)} = a \) (constant)

\( t \geq 0 \)

From (1) \( x(0) = x_0 \)

\( y(0) = x_0 \) since \( y(t) \) is also a solution.
\[
\frac{y(0)}{
\left.\frac{\partial y}{\partial x} \right|_{x=0}} = \frac{x_0}{X_0 e^{x_0}} = \frac{x_0}{x_0} = 1 = a
\]

\[
\Rightarrow \quad \frac{\partial y}{\partial x}(t) = a = 1 \Rightarrow y(t) = x(t)
\]

so the solution
\[
x(t) = x_0 e^{at}
\]

is unique.

---

Now we can use this to solve transistors circuits with RC model.

---

![Logic circuit diagram](https://via.placeholder.com/150)

---

![CMOS circuit diagram](https://via.placeholder.com/150)

---

\[
G = \frac{1}{RC} \quad \Rightarrow \quad \frac{1}{C} \quad \Rightarrow \quad \frac{1}{R}
\]

\[ t < 0, \quad V_{\text{in}} = 0V \]

\[ t \geq 0, \quad V_{\text{in}} = V_{\text{dd}} \]

**KCL:** \[ I_1 + I_2 + I_3 = 0 \]

**Elements:**
- \[ V_1 = I_1 \cdot R_{\text{on,m1}} \]
- \[ I_2 = C_{\text{gn2}} \frac{dV_2}{dt} \]
- \[ I_3 = C_{\text{gp2}} \frac{dV_3}{dt} \]

**Voltages:**
- \[ V_1 = V_X \]
- \[ V_2 = V_X \]
- \[ V_3 = V_X - V_{\text{dd}} \]
Sub. into KCL:

\[
\frac{V_1}{R_{on,m1}} + C_{m2} \frac{dV_1}{dt} + C_{p2} \frac{dV_3}{dt} = 0
\]

\[
\frac{V_x}{R_{on,m1}} + C_{m2} \frac{dV_x}{dt} + C_{p2} \frac{d}{dt}(V_x - V_{dd}) = 0
\]

\[
\frac{V_x}{R_{on,m1}} + C_{m2} \frac{dV_x}{dt} + C_{p2} \frac{d}{dt} V_x = 0
\]

\[
\frac{V_x}{R_{on,m1}} + ((C_{m2} + C_{p2}) \frac{d}{dt} V_x = 0
\]

\[
\frac{d}{dt} V_x(t) = -\frac{V_x}{R_{on,m1} \cdot (C_{m2} + C_{p2})}
\]

\[
T = R_{on,m1} \cdot (C_{m2} + C_{p2})
\]

determines the speed of transition.

Already have a solution for this:

\[
t > 0: \quad V_x(t) = V_x(0) e^{-\frac{t}{T}}
\]

\[
V_x(0) = V_{dd} \Rightarrow V_x(t) = V_{dd} e^{-\frac{t}{T}}
\]