This homework is due on Thursday, October 22, 2020, at 10:59PM.

Self-grades are due on Thursday, October 29, 2020, at 10:59PM.

1 LED Strip

I have an LED strip with 5 red LEDs whose brightnesses I want to set. These LEDs are addressed as a queue: at each time step, I can push a new brightness command between 0 and 255 to the left-most LED. Each of the following LEDs will then take on the brightness previously displayed by the LED immediately to its left.

a) What should we use for our state vector? What does it mean that this is a state vector? What is our input?

Solution

We can use the brightnesses of each LED as our state vector. We can use these values as our state vector since together with the input, they describe everything about our system that we need to know in order to predict what our system will do in the future. Our input is the command to the left-most LED.

b) Assume that our system is linear, and write out the state equations in matrix form. Please choose a reasonable order for the state variables in the state vector.

Solution

The system can be written in the form $\vec{x}(t+1) = A\vec{x}(t) + Bu$. Ordering the LED brightnesses in the state vector from left to right, we get:

$$\vec{x}(t+1) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t)$$

If you chose to put the left-most LED's brightness last in the state vector (so that the LEDs are ordered right to left and the state vector gets flipped upside down), the A matrix gets transposed and the B matrix is flipped upside down.

c) Is this system controllable? Explain intuitively what this system's controllability means in terms of LED brightnesses.

Solution

Testing for controllability we see:

$$\begin{bmatrix} A^4B & A^3B & A^2B & AB & B \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

which has full rank. This means that the system is controllable. A system is called controllable if from any initial state, we can reach any final state that we desire at some time in the future.

For our LED strip, controllability means that we can display any set of brighnesses that we desire, but it may take a few time steps to get there.

d) Starting from the pattern of brightnesses (from left to right) [0, 127, 0, 255, 0], can we maintain this pattern for all future time steps? Can we display any fixed pattern of brightnesses for all time?

Solution

We cannot display [0, 127, 0, 255, 0] for all time. Immediately after we display this set of brightnesses, we will display [u(1), 0, 127, 0, 255].

If we want to display a fixed and unchanging set of brightnesses, every element in our state vector must be the same.

Controllability tells us only that we can *reach* any desired state (sometimes only temporarily). It does not mean we can *keep* our system at any desired state for all time.

2 Controllability in circuits

Consider the circuit in Figure 1, where V_s is an input we can control:

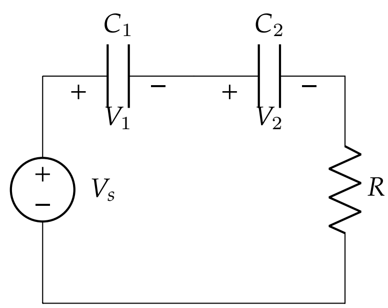


Figure 1: Controllability in circuits

a) Write the state space model for this circuit.

Solution

$$I = \frac{V_s - V_1 - V_2}{R} = C_1 \frac{dV_1}{dt} = C_2 \frac{dV_2}{dt}$$

$$\frac{d}{dt}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC_1} & -\frac{1}{RC_1} \\ -\frac{1}{RC_2} & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{RC_1} \\ \frac{1}{RC_2} \end{bmatrix} V_s$$

b) Show that this system is not controllable.

Solution

If we calculate AB, we find that it is a linear combination of B:

$$AB = \begin{bmatrix} -\frac{1}{RC_1}(\frac{1}{RC_1} + \frac{1}{RC_2}) \\ -\frac{1}{RC_2}(\frac{1}{RC_1} + \frac{1}{RC_2}) \end{bmatrix} = -(\frac{1}{RC_1} + \frac{1}{RC_2})B$$

This means that the controllability matrix

$$\begin{bmatrix} AB & B \end{bmatrix}$$

Must have rank of 1. Therefore, this system is not controllable.

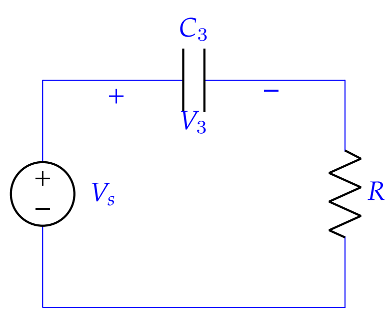
c) Explain, in terms of circuit currents and voltages, why this system isn't controllable. (Hint: think about what currents/voltages of the circuit we are controlling with V_s)

Solution

We can only control V_s, which in turn controls the amount of current flowing through the circuit. Since this current is equal through both capacitors and current directly affects the voltage across a capacitor, there is no way to individually control the voltages across the capacitors.

d) Draw an equivalent circuit of this system that is controllable. What quantity can you control in this system?

Solution



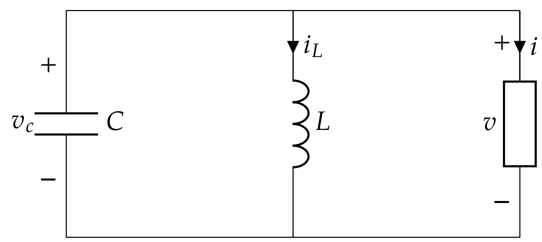
We can control V_3 in this circuit.

3 Nonlinear circuit component

This is a problem adapted from a past midterm problem (Spring 2017 midterm 2).

Consider the circuit below that consists of a capacitor, inductor, and a third element with a nonlinear voltage-current characteristic:

$$i = 2v - v^2 + 4v^3$$



a) Write a state space model of the form

$$\frac{dx_1(t)}{dt} = f_1(x_1(t), x_2(t))$$

$$\frac{dx_2(t)}{dt} = f_2(x_1(t), x_2(t))$$

Where $x_1(t) = v_c(t)$ and $x_2(t) = i_L(t)$.

Solution

We need to get $\frac{dv_c}{dt}$ and $\frac{di_L}{dt}$ in terms of v_c and i_L.
All the components are in parallel, so:

$$v_c = v_L = v$$

Using the relation of an inductor's current and voltage:

$$v_c = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{1}{L}v_c \tag{1}$$

Using KCL, we can say:

$$i_c + i_L + i = 0$$

$$C\frac{dv_c}{dt} + i_L + 2v - v^2 + 4v^3 = 0$$

$$\frac{dv_c}{dt} = \frac{1}{C}\left(-i_L - 2v_c + v_c^2 - 4v_c^3\right) \tag{2}$$

Taking equations (1) and (2) and substituting in x_1 and x_2 gives us our answer:

$$\frac{dx_1}{dt} = f_1(x_1, x_2) = \frac{1}{C}\left(-x_2 - 2x_1 + x_1^2 - 4x_1^3\right)$$

$$\frac{dx_2}{dt} = f_2(x_1, x_2) = \frac{1}{L}x_1$$

b) Linearize the state model at the equilibrium point $x_1 = x_2 = 0$ and specify the resulting A matrix.

Solution

$$A = \nabla f(\vec{x})\Big|_{x_1=x_2=0} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}\Bigg|_{x_1=x_2=0} = \begin{bmatrix} \frac{1}{C}\left(-2+2x_1-12x_1^2\right) & -\frac{1}{C} \\ \frac{1}{L} & 0 \end{bmatrix}\Bigg|_{x_1=x_2=0}$$

$$A = \begin{bmatrix} -\frac{2}{C} & -\frac{1}{C} \\ \frac{1}{L} & 0 \end{bmatrix}$$

c) Is the linearized system stable?

Solution

$$\det(A-\lambda I) = \lambda^2 + \frac{2}{C}\lambda + \frac{1}{LC} = 0$$

$$\lambda = \frac{-\frac{2}{C} \pm \sqrt{\left(\frac{2}{C}\right)^2 - \frac{4}{LC}}}{2}$$

For a continuous system to be stable, all eigenvalues must have $\mathrm{Re}\{\lambda\} < 0$.

Since both L and C can only take positive values, the square root term will always have a real part smaller than $\frac{2}{C}$, which means both eigenvalues will have negative real parts. The system is stable.

4 An Extension of Predator-Prey Models

On last week's homework, we looked at a model of the predator-prey dynamic in biological systems, shown below:

$$\frac{d}{dt}x(t) = (a-by)x \tag{1}$$

$$\frac{d}{dt}y(t) = (cx-d)y \tag{2}$$

This question will seek to expand on that model to create a more generalized version for systems of more than just two species. Specifically, we will look at the *Generalized Lotka-Volterra Equations*:

$$\frac{d}{dt}x_i(t) = x_i\left(b_i + \sum_{j=1}^{n} a_{ij}x_j\right), i = 1, \dots, n \tag{3}$$

Where each x_i is the density of species i in the population, b_i is the species' growth rate, and a_{ij} are interaction parameters between species i and species j.

a) Show that, if $A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$ is invertible, there are at most 2^n different equilibrium points for the n-species system described by these a_{ij}.

Solution

Note that, for a system to be an equilibrium point, it must satisfy $\frac{d}{dt}x_i(t) = 0$ for all i. Therefore we have

$$0 = x_i\left(b_i + \sum_{j=1}^{n} a_{ij}x_j\right), i = 1, \ldots, n \tag{4}$$

By the Zero-Product Property, we have

$$0 = x_i \text{ OR } 0 = \left(b_i + \sum_{j=1}^{n} a_{ij}x_j\right), i = 1, \ldots, n \tag{5}$$

Now, at this point, our intent was for you to conclude that, with n equations, each of which has 2 possible options, you would have 2^n total possibilities.

However, many of you raised a valid point that there could be infinitely many solutions, if A is not invertible. We can see this by choosing all $x_i \neq 0$ (as though we take the second option for all i). Then, we have the standard linear system, which we can rephrase as $A\vec{x} = \vec{b}$. If A is not full rank, then there is a nullspace here that we can abuse to find infinitely many solutions, if there exist any solutions $\vec{x}$.

However, it is not enough for A to be full rank – all of A's submatrices must also be. For example, consider the case of $n = 3$ below:

$$\frac{d}{dt}x_1(t) = x_1\left(b_1 + \sum_{j=1}^{3} a_{1j}x_j\right) = 0 \tag{6}$$

$$\frac{d}{dt}x_2(t) = x_2\left(b_2 + \sum_{j=1}^{3} a_{2j}x_j\right) = 0 \tag{7}$$

$$\frac{d}{dt}x_3(t) = x_3\left(b_3 + \sum_{j=1}^{3} a_{3j}x_j\right) = 0 \tag{8}$$

Let's further suppose $A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$. Then, if we set $x_3 = 0$ to satisfy the third differential equation above, we can reduce the other two down:

$$\frac{d}{dt}x_1(t) = x_1\left(b_1 + \sum_{j=1}^{2} a_{1j}x_j\right) = 0 \tag{9}$$

$$\frac{d}{dt}x_2(t) = x_2\left(b_2 + \sum_{j=1}^{2} a_{2j}x_j\right) = 0 \tag{10}$$

$$\frac{d}{dt}x_3(t) = 0 \tag{11}$$

Note the change in upper bounds on the first two summations – if $x_3 = 0$, we do not need to consider adding $a_{13}x_3x_1$, as this is also 0. You may notice that the first two equations now look exactly like the case where $n = 2$; however, my new A matrix for the reduced case is $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$. This is not invertible, which poses a problem for us by the above line of reasoning.

It turns out, that we must have all of these submatrices be invertible for this system to have precisely 2^n solutions – an equivalent condition here would have been to state that the matrix A has strictly positive eigenvalues – the reasoning why, however, is out of scope for this course. This last bit of nuance, from recognizing that A must be invertible onwards, is not something that we will expect you to recognize on any sort of exam.

b) What does it mean for an a_{ij} to be positive? Negative? Zero?

Solution

The a_{ij} terms correspond to a sort of interaction parameter in the system; species j can either support the survival of species i, or it can compete or leech off of it somehow. In each of these cases, we would expect a_{ij} to be positive (having more of species j promotes the growth of species i), or negative (having more of species j keeps species i from growing).

c) Let $n = 2$. Assume that a_{11} and a_{22} are both nonzero, b_1 and b_2 are both greater than zero, and that the matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is invertible. Under these assumptions, there are up to 4 equilibrium points in this system; what are they?

Solution

There are 4 possible scenarios here:, corresponding to each choice of the above situation ($0 = x_i$ OR $0 = \left(b_i + \sum_{j=1}^{n} a_{ij}x_j\right)$):

- $x_1 = 0, x_2 = 0$
- $x_1 = 0, b_2 + a_{21}x_1 + a_{22}x_2 = 0$
- $b_1 + a_{11}x_1 + a_{12}x_2 = 0, x_2 = 0$
- $b_1 + a_{11}x_1 + a_{12}x_2 = 0, b_2 + a_{21}x_1 + a_{22}x_2 = 0$

Case (1): this system has solved itself—the corresponding point is $\vec{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Case (2): in this system, we have $x_1 = 0$. Therefore we have $b_2 + a_{21}(0) + a_{22}x_2 = 0 \implies x_2 = -\frac{b_2}{a_{22}}$.

Case (3): in this system, we have $x_2 = 0$. Therefore we have $b_1 + a_{11}x_1 + a_{12}(0) = 0 \implies x_1 = -\frac{b_1}{a_{11}}$.

Case (4): in this system, we have two linear equations in two unknowns. We can set this up in a matrix and solve the system of equations to find that $\vec{x} = A^{-1}\vec{b}$ for A defined above.

d) One of these points is of the form $\vec{x}^* = \begin{bmatrix} 0 \\ x_2^* \end{bmatrix}$, with nonzero x_2. Linearize the system about this point.

Solution

We know from lecture that a continuous-time system with n variables can be represented as

$$\frac{d}{dt}\vec{x}(t) = f(\vec{x}(t)) \tag{12}$$

From Taylor's Theorem we can say

$$f(\vec{x}) \approx f(\vec{x}^*) + \nabla f(\vec{x}^*)(\vec{x} - \vec{x}^*) \tag{13}$$

In our example, the function f is given by

$$f(\vec{x}) = \begin{bmatrix} b_1 x_1 + a_{11}x_1^2 + a_{12}x_1x_2 \\ b_2x_2 + a_{21}x_1x_2 + a_{22}x_2^2 \end{bmatrix} \tag{14}$$

So, given our point from before $\left(\vec{x}^* = \begin{bmatrix} 0 \\ -b_2/a_{22} \end{bmatrix}\right)$, we have

$$f(\vec{x}^*) = \vec{0} \tag{15}$$

$$\nabla f(\vec{x}) = \begin{bmatrix} b_1 + 2a_{11}x_1 + a_{12}x_2 & a_{12}x_1 \\ a_{21}x_2 & b_2 + a_{21}x_1 + 2a_{22}x_2 \end{bmatrix} \tag{16}$$

$$\Rightarrow \nabla f(\vec{x}^*) = \begin{bmatrix} b_1 - b_2\left(\frac{a_{12}}{a_{22}}\right) & 0 \\ -b_2\left(\frac{a_{21}}{a_{22}}\right) & -b_2 \end{bmatrix} \tag{17}$$

So, our linearized system will look like:

$$f(\vec{x}) \approx \begin{bmatrix} b_1 - b_2\left(\frac{a_{12}}{a_{22}}\right) & 0 \\ -b_2\left(\frac{a_{21}}{a_{22}}\right) & -b_2 \end{bmatrix}\left(\vec{x} - \begin{bmatrix} 0 \\ -\frac{b_2}{a_{22}} \end{bmatrix}\right) \tag{18}$$

e) Under what conditions on A and/or $\vec{b}$ is this point stable? Does this make sense in the context of population dynamics?

Solution

In order for the system to be stable, we need the eigenvalues of the matrix of our linearized system to be stable. Given that our matrix was determined to be: $\begin{bmatrix} b_1 - b_2\left(\frac{a_{12}}{a_{22}}\right) & 0 \\ -b_2\left(\frac{a_{21}}{a_{22}}\right) & -b_2 \end{bmatrix}$, we can find that our eigenvalues are:

$$\lambda_1 = b_1 - b_2\left(\frac{a_{12}}{a_{22}}\right), \lambda_2 = -b_2 \tag{19}$$

We are given that $b_1 > 0$, $b_2 > 0$ – therefore, in order for this system to be stable, we need $b_1 - b_2\left(\frac{a_{12}}{a_{22}}\right) < 0$. Rewriting this:

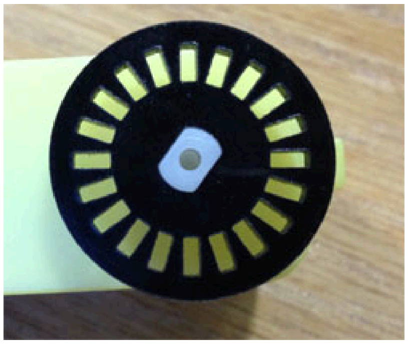
$$b_1 - b_2\left(\frac{a_{12}}{a_{22}}\right) < 0 \tag{20}$$

$$\frac{b_1}{a_{12}} < \frac{b_2}{a_{22}} \tag{21}$$

In other words, we want the ratio of species 2's growth rate to its interactions with itself to be greater than the ratio of species 1's growth rate to its interactions with species 2. For this point to be stable, we must be able to introduce a small amount of species 1, and observe that species 1 naturally dies off to return back to this original population total. Therefore, we should expect that for this to be stable, either species 2 must antagonize species 1, or species 1 must not grow quickly; this is reflected here, as for this inequality to hold, at least one of these two statements must be true (species 2 antagonizing species 1 $\longrightarrow a_{12} < 0$, making that whole term negative; alternatively, species 1 grows slowly $\longrightarrow b_1 < b_2$).

5 Understanding the SIXT33N Car Control Model

As the students in the lab continue along the process of making the awesome SIXT33N cars, we'd like to better understand the car model that they will be using to develop a control scheme. As a wheel on the car turns, there is an encoder disc (see below) that also turns as the wheel turns. The encoder shines a light though the encoder disc, and as the wheel turns, the light is continually blocked and unblocked, allowing the encoder to detect how fast the wheel is turning by looking at the number of times that the light "ticks" between being blocked and unblocked over a specific time interval.



The following model applies separately to each wheel (and associated motor) of the car:

$$v[k] = d[k+1] - d[k] = \theta u[k] - \beta$$

Meet the variables at play in this model:

- k - The current timestep of the model. Since we model the car as a discrete system, this will advance by 1 on every new sample in the system.
- $d[k]$ - The total number of ticks advanced by a given encoder (the values may differ for the left and right motors–think about when this would be the case).
- $v[k]$ - The discrete-time velocity (in units of ticks/timestep) of the wheel, measured by finding the difference between two subsequent tick counts ($d[k+1] - d[k]$).
- $u[k]$ - The input to the system. The motors that apply force to the wheels are driven by an input voltage signal. This voltage is delivered via a technique known as pulse width modulation (PWM), where the average value of the voltage (which is what the motor is responsive to) is controlled by changing the duty cycle of the voltage waveform. The duty cycle, or percentage of the square wave's period for which the square wave is HIGH, is mapped to the range $[0, 255]$. Thus, $u[k]$ takes a value in $[0, 255]$ representing the duty cycle. For example, when $u[k] = 255$, the duty cycle is 100 %, and the motor controller just delivers a constant signal at the system's HIGH voltage, delivering the maximum possible power to the motor. When $u[k] = 0$, the duty cycle is 0 %, and the motor controller delivers 0 V.

- θ - Relates change in input to change in velocity: if the wheel rotates through n ticks in one timestep for a given $u[k]$ and m ticks in one timestep for an input of $u[k]+1$, then $\theta = m - n = \frac{\Delta v[k]}{\Delta u[k]} = \frac{v_{u_1[k]}[k] - v_{u_0[k]}[k]}{u_1[k] - u_0[k]}$. **Its units are ticks/(timestep · duty cycle).** Since our model is linear, we assume that θ is the same for every unit increase in $u[k]$. This is empirically measured using the car: θ depends on many physical phenomena, so for the purpose of this class, we will not attempt to create a mathematical model based on the actual physics. However, you can conceptualize θ as a "sensitivity factor", representing the idiosyncratic response of your wheel and motor to a change in power (you will have a separate θ for your left and your right wheel).
- β - Similarly to θ, β is dependent upon many physical phenomena, so we will empirically determine it using the car. β represents a constant offset in velocity, and hence **its units are ticks/timestep**. Note that you will also have a different β for your left and your right wheel.

In this problem (except parts (c) and (e)) we will assume that the wheel conforms perfectly to this model to get an intuition of how the model works.

a) If we wanted to make the wheel move at a certain target velocity v^*, what input $u[k]$ should we provide to the motor that drives it? Your answer should be symbolic, and in terms of v^*, $u[k]$, θ, and β.

Solution

$$v^* = \theta u[k] - \beta$$
$$v^* + \beta = \theta u[k]$$
$$u[k] = \frac{v^* + \beta}{\theta}$$

b) Even if the wheel and the motor driving it conform perfectly to the model, our inputs still limit the range of velocities. Given that $0 \le u[k] \le 255$, determine the maximum and minimum velocities possible for the wheel. How can you slow the car down?

Solution

The maximum is $255\theta - \beta$, and the minimum is $0 - \beta = -\beta$.

Since there are no brakes on the wheel, we slow down by reducing the input value and thereby the PWM duty cycle.

c) Our intuition tells us that a wheel on a car should eventually stop turning if we stop applying any power to it. Find $v[k]$ assuming that $u[k] = 0$. Does the model obey your intutition? What does that tell us about our model?

Solution

$$v[k] = -\beta$$

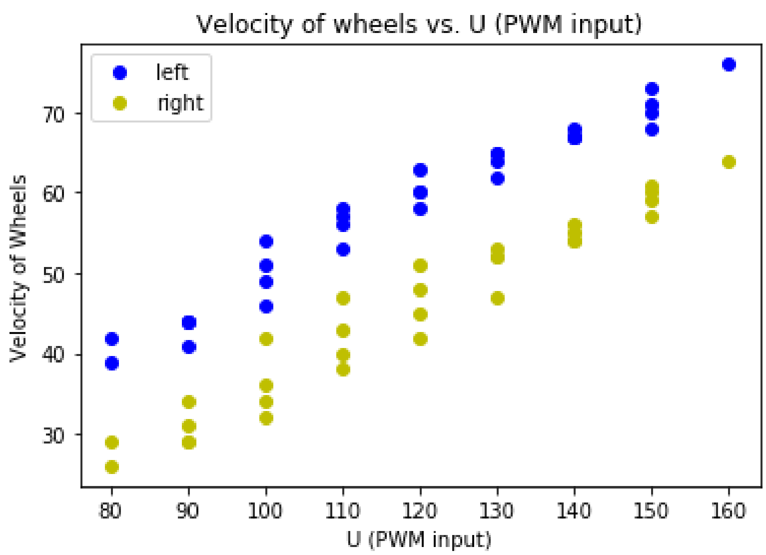
However, our intuition says that the car should have stopped: i.e., $v[k] = 0$. In lab, we will empirically find the value of β over a range of input duty cycles, but our fit does not work very well everywhere and our model does not match the real behavior near $u = 0$. This is a limitation of the simplified empirical model we are using, but as we will see in lab, we can still make the actual car work well over a certain range of inputs.

d) In order to characterize the car, we need to find the θ and β values that model your left and right wheels and their motors: θ_l, β_l, θ_r, and β_r. How would you determine θ and β empirically? What data would you need to collect? *Hint:* keep in mind we also know the input $u[k]$ for all k.

Solution

Given the motor model $v[k] = d[k+1] - d[k] = \theta u[k] - \beta$, we can determine θ and β by plotting velocity ($v[k]$) vs. input ($u[k]$).
By sweeping the input over a reasonable operating range and collecting multiple velocity samples (by differencing the total number of ticks, a collected quantity, between subsequent timesteps), we can collect velocity and input data. Then, we can perform least-squares linear regression on the data to determine the slope θ and y-intercept β.

Least-squares example solution:



Using the least squares linear regression method shown below, we can estimate the slope and y-intercept for each graph.

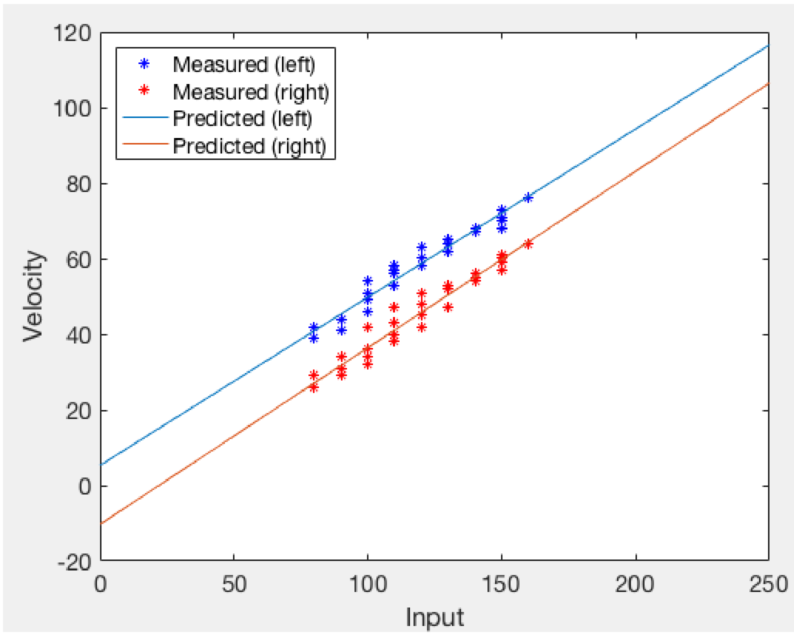
$$A^T A\hat{\boldsymbol{x}} = A^T \boldsymbol{y} \tag{22}$$

$$A = \begin{bmatrix} u_0 & 1 \\ u_1 & 1 \\ \vdots & \vdots \\ u_n & 1 \end{bmatrix} \tag{23}$$

$$y = \begin{bmatrix} v_{\text{measured},0} \\ v_{\text{measured},1} \\ \vdots & \vdots \\ v_{\text{measured},n} \end{bmatrix} \tag{24}$$

Using the θ and β values we found above, we now plot the predicted velocities of the left and right motor over the inputs u from 0 to 255:

$$v_{\text{left}} = \theta_{\text{left}} u - \beta_{\text{left}} \tag{25}$$
$$v_{\text{right}} = \theta_{\text{right}} u - \beta_{\text{right}} \tag{26}$$



e) How can you use the data you collected in part (d) to mitigate the effect of the system's nonlinearity and/or minimize model mismatch? *Hint:* you will only wind up using a small range of the possible input values in practice. There are several reasons this is true, but one is that each motor has a characteristic attainable velocity range, and for your car to drive straight, we need the wheels to rotate with the same velocity.

Solution

Since we model the relationship between input and velocity as linear, if the actual data we collect displays nonlinearity, the car's behavior will not match the model well. To minimize model mismatch, we choose an operating point centered within an approximately linear region of the plot, *as long as both motors are capable of attaining a reasonable amount of the surrounding velocities.*

6 Homework Process and Study Group

Citing sources and collaborators are an important part of life, including being a student!
We also want to understand what resources you find helpful and how much time homework is taking, so we can change things in the future if possible.

a) **What sources (if any) did you use as you worked through the homework?**

b) **If you worked with someone on this homework, who did you work with?**
List names and student ID's. (In case of homework party, you can also just describe the group.)

c) **Roughly how many total hours did you work on this homework?**

d) **Do you have any feedback on this homework assignment?**