

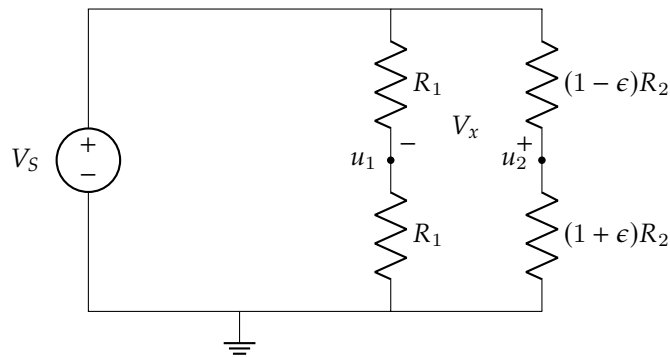
**This homework is due on Thursday, September 10, 2020, at 10:59PM.  
Self-grades are due on Thursday, September 17, 2020, at 10:59PM.**

## 1 MEMS Accelerometer

A micro-electromechanical system (MEMS) accelerometer is a device that can measure acceleration, for example by using a set of strain-sensitive resistors. There are three in every cell phone, detecting the phone's orientation and motion. MEMS accelerometers are made using silicon micromachining. In the accelerometer, a silicon block with a known mass is suspended between springs made of silicon. The compression of the springs can be measured because the resistance of a silicon spring changes when it is compressed. (This occurs because silicon is a piezoresistive material, which we will not talk about in this course.)

Accelerating the device causes the silicon block to move, changing the compression of the attached silicon springs, and therefore changing the resistance across the springs. One of the springs will be compressed while another will be extended, so the resistance of one spring increases while the other decreases. If we measure the changes in the resistance of the springs, then we can understand how the silicon block is moving.

However, the change in resistance is extremely small. For instance, for a change of  $9.8\text{m/s}^2$  of acceleration (equivalent to the Earth's gravitational acceleration,  $g$ ), the resistance only changes by about 4% in our example! To measure such a small resistance change, the resistors are placed in the following configuration known as a Wheatstone bridge:



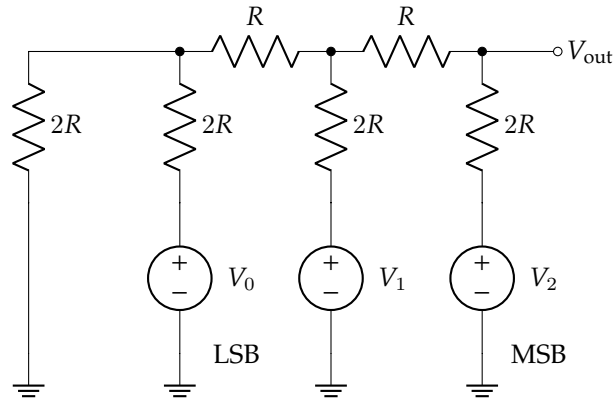
The resistances of the two resistors on the left, both with the same value  $R_1$ , will remain constant. The two resistances on the right represent the silicon springs. The  $\epsilon$  term represents the fractional change in resistance brought about by movement of the silicon block. For example, if  $\epsilon = 0.01$ , then the resistance of spring being compressed will increase by 1%, while the resistance of the spring being extended will decrease by 1%.

A voltmeter measures the voltage difference  $V_x = u_2 - u_1$  on the device. We use  $V_x$  to determine the change in resistance and hence the acceleration.

- To determine the acceleration, we first need to understand the relationship between our measured voltage  $V_x$  and the resistances of the springs. **What is  $V_x$  in terms of  $R_1$ ,  $R_2$ ,  $\epsilon$ , and  $V_S$ ?**
- Suppose the minimum voltage the voltmeter can detect is  $V_x = 1\mu\text{V}$ . If this is the minimum  $V_x$ , **what is the minimum measurable resistance change  $\epsilon$  that we can measure?** We are going to make the simplifying assumption that the  $\epsilon$  varies linearly with the acceleration. If each acceleration change of  $9.8\text{m/s}^2$  ( $1g$ ) corresponds to a change in resistance  $\epsilon = 0.04$ , then **what is the minimum acceleration that can be measured by this system?** The answer may be expressed in terms of  $V_S$ .

## 2 Digital-Analog Converter

A digital-analog converter (DAC) is a circuit for converting a digital representation of a number (binary) into a corresponding analog voltage. In this problem, we will consider a DAC made out of resistors only (resistive DAC) called the  $R$ - $2R$  ladder. Here is the circuit for a 3-bit resistive DAC.



Let  $b_0, b_1, b_2 = \{0, 1\}$  (that is, either 1 or 0), and let the voltage sources  $V_0 = b_0 V_{DD}$ ,  $V_1 = b_1 V_{DD}$ ,  $V_2 = b_2 V_{DD}$ , where  $V_{DD}$  is the supply voltage.

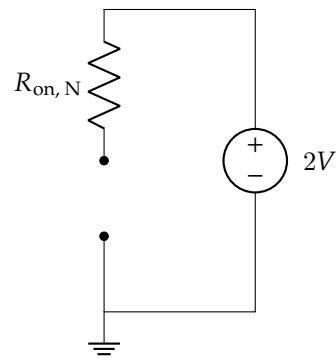
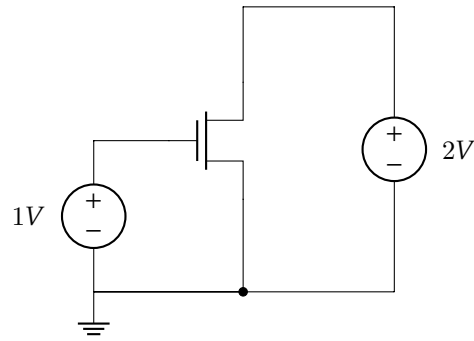
As you may have noticed,  $(b_2, b_1, b_0)$  represents a 3-bit binary (unsigned) number where each of  $b_i$  is a binary bit. We will now analyze how this converter functions.

- If  $b_2, b_1, b_0 = 1, 0, 0$ , what is  $V_{out}$ ? Express your answer in terms of  $V_{DD}$ .
- If  $b_2, b_1, b_0 = 0, 1, 0$ , what is  $V_{out}$ ? Express your answer in terms of  $V_{DD}$ .
- If  $b_2, b_1, b_0 = 0, 0, 1$ , what is  $V_{out}$ ? Express your answer in terms of  $V_{DD}$ .
- If  $b_2, b_1, b_0 = 1, 1, 1$ , what is  $V_{out}$ ? Express your answer in terms of  $V_{DD}$ . (*Hint*: you shouldn't have to do too much work here, if you've already completed parts (a)-(c).)
- Finally, solve for  $V_{out}$  in terms of  $V_{DD}$  and the binary bits  $b_2, b_1, b_0$ .
- Explain how your results above show that the resistive DAC converts the 3-bit binary number  $(b_2, b_1, b_0)$  to the output analog voltage  $V_{out}$ .

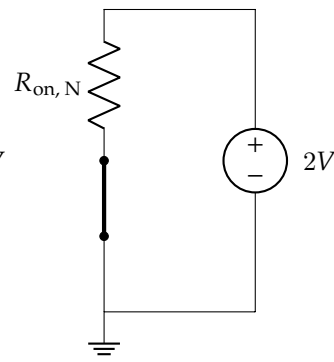
## 3 Transistor Behavior

For all NMOS devices in this problem,  $V_{tn} = 0.5V$ . For all PMOS devices in this problem,  $|V_{tp}| = 0.6V$ .

- Which is the equivalent circuit for the right-hand side of the circuit? **Fill in the correct bubble.**



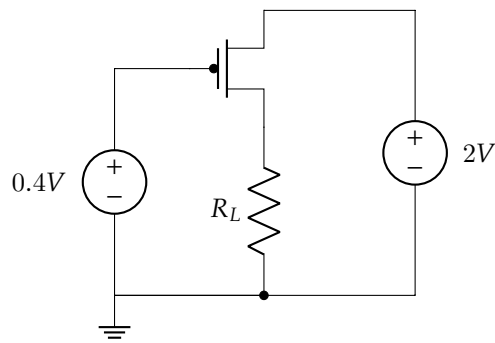
Circuit A

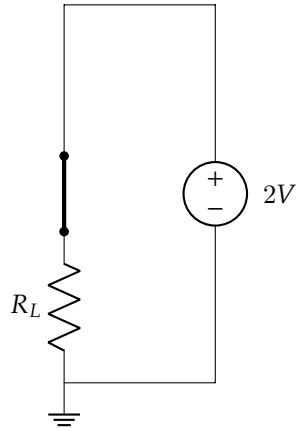


Circuit B

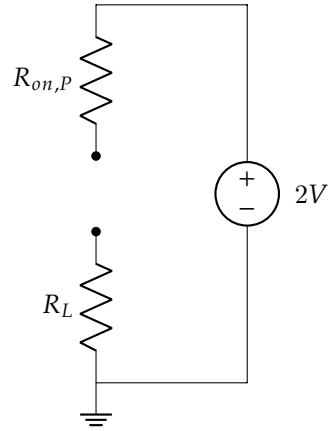
	A	B
Equivalent Circuit	<input type="radio"/>	<input type="radio"/>

b) Which is the equivalent circuit for the right-hand side of the circuit? **Fill in the correct bubble.**

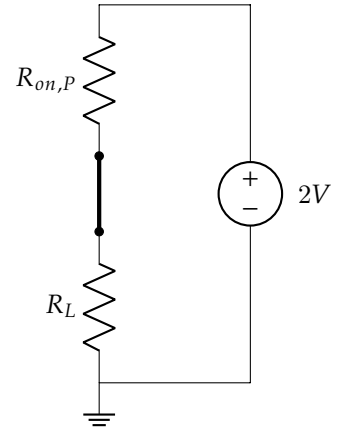




Circuit A



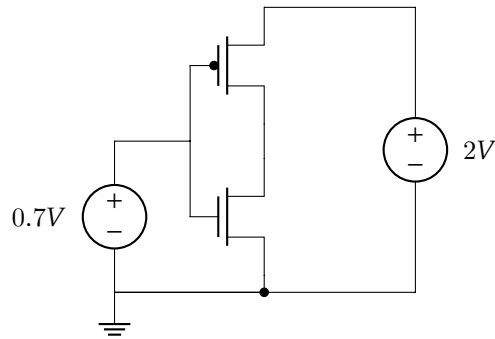
Circuit B

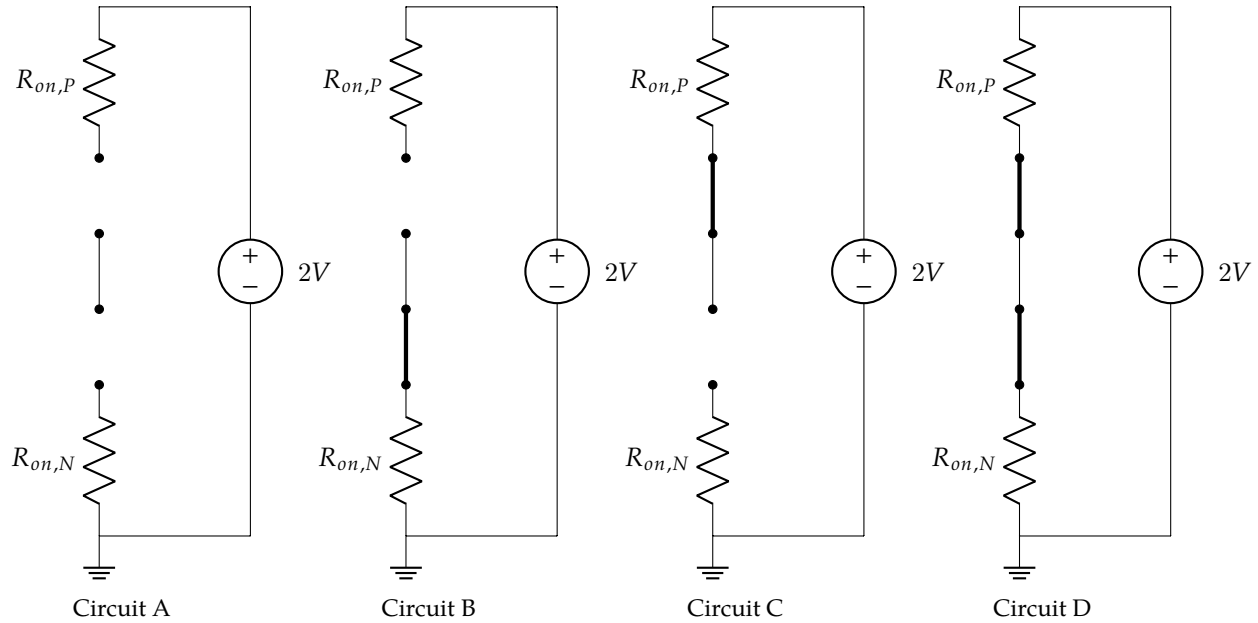


Circuit C

	A	B	C
<b>Equivalent Circuit</b>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

c) Which is the equivalent circuit for the right-hand side of the circuit? **Fill in the correct bubble.**



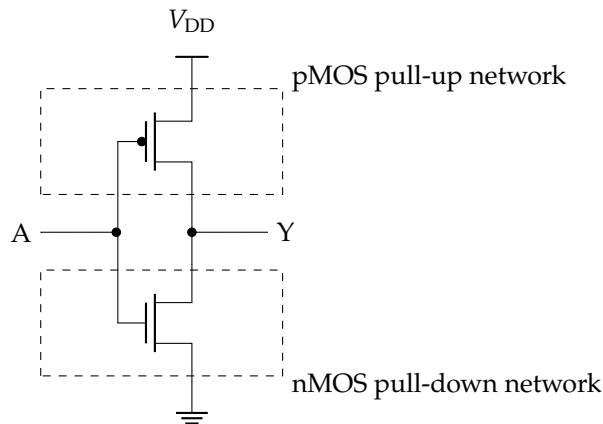


	A	B	C	D
Equivalent Circuit	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

### 4 Transistors and Boolean Logic

A boolean formula can be implemented in digital circuitry using nMOS and pMOS transistors. In circuits, the truth value 1 (*true*) is represented by a high voltage, called POWER ( $V_{DD}$ ). The truth value 0 (*false*) is represented by a low voltage, called GROUND (GND). In this problem, we will only use the truth values in order to simplify notations. That is, if you see  $A = 1$  for a point  $A$ , then it means the voltage of  $A$  is equal to  $V_{DD}$ . Similarly, if  $A = 0$ , then the voltage of  $A$  is equal to GND.

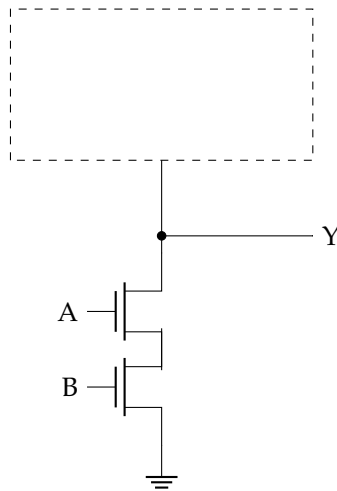
An inverter can be implemented with 1 nMOS and 1 pMOS, as shown in the figure below. When the input  $A$  is 0, then the nMOS is OFF and the pMOS is ON. Thus, the output  $Y$  is pulled up to 1 because it is connected to  $V_{DD}$ . Conversely, when  $A$  is 1, then the nMOS is ON and the pMOS is OFF, and  $Y$  is pulled down to 0. Therefore, the circuit implements the Boolean formula,  $Y = \bar{A}$ .



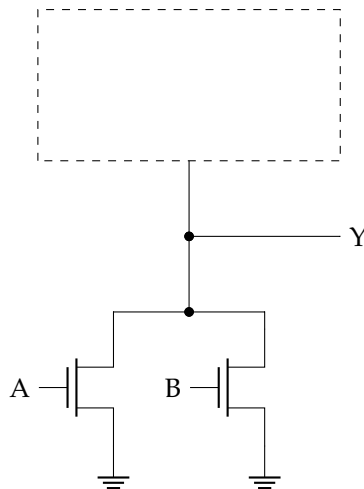
In general, a Boolean-formula circuit has an nMOS *pull-down network* to connect the output to 0 (GND) and a pMOS *pull-up network* to connect the output to 1 ( $V_{DD}$ ). The pull-up and pull-down networks in the inverter example each consist of a single transistor. Note that, when the input  $A$  is 0, no current flows through the circuit as depicted above; likewise for when the input  $A$  is 1. This is a hallmark of good design for pull-up and pull-down networks: the network should never, at any point or state, have any current flowing through the circuit diagram as depicted. This way, we can minimize the power dissipated in this circuit – if current were flowing, we would be wasting power. There may be other components that do draw current from the circuit, but we will not worry about that in this question.

In this problem, you will design pull-up networks given the pull-down networks.

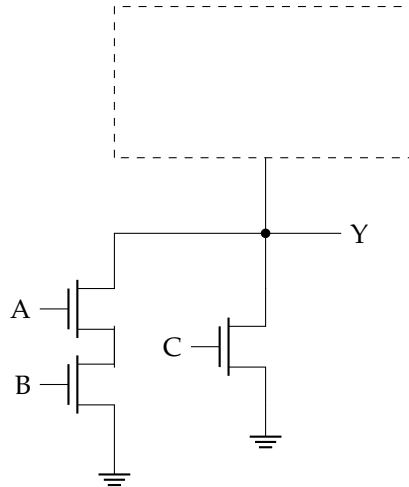
- a) The pull-down network of the Boolean formula (a 2-input NAND gate),  $Y = \overline{A \cdot B}$ , is given below. Design the pull-up network (the dashed box) with 2 pMOS transistors.



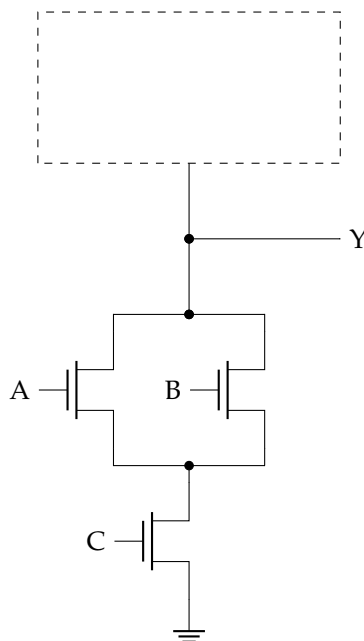
- b) The pull-down network of the Boolean formula (a 2-input NOR gate),  $Y = \overline{A + B}$ , is given below. Design the pull-up network (the dashed box) with 2 pMOS transistors.



- c) The pull-down network of the Boolean formula,  $Y = \overline{((A \cdot B) + C)}$ , is given below. Design the pull-up network (the dashed box) with 3 pMOS transistors.

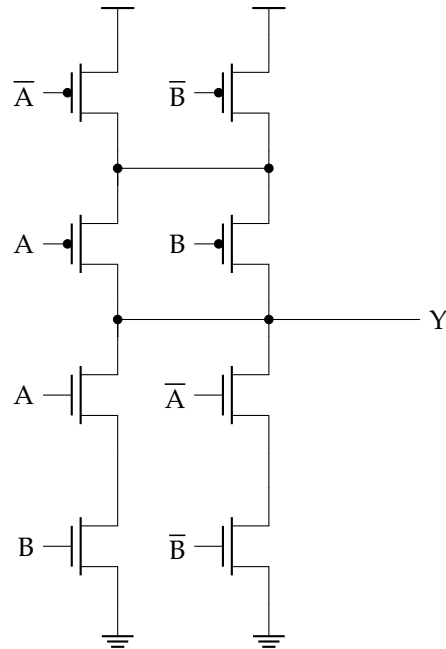


- d) The pull-down network of the Boolean formula,  $Y = \overline{((A + B) \cdot C)}$ , is given below. Design the pull-up network (the dashed box) with 3 pMOS transistors.



- e) For the circuit below, write the truth table for inputs A and B with output Y. What boolean operation is this?

Note some of the gate voltages are  $\bar{A}$  and  $\bar{B}$ .



## 5 Homework Process and Study Group

Citing sources and collaborators are an important part of life, including being a student! We also want to understand what resources you find helpful and how much time homework is taking, so we can change things in the future if possible.

- What sources (if any) did you use as you worked through the homework?**
- If you worked with someone on this homework, who did you work with?**  
List names and student ID's. (In case of homework party, you can also just describe the group.)
- Roughly how many total hours did you work on this homework?**
- Do you have any feedback on this homework assignment?**