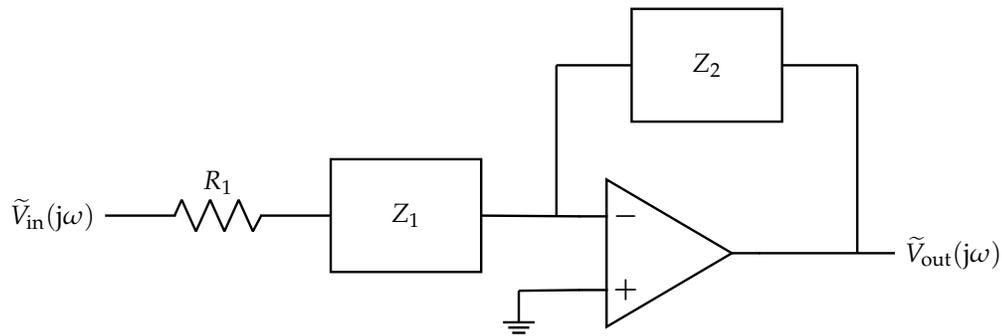


Homework 5

This homework is due on Saturday, February 24, 2024, at 11:59PM.

1. Circuit Design Part 2

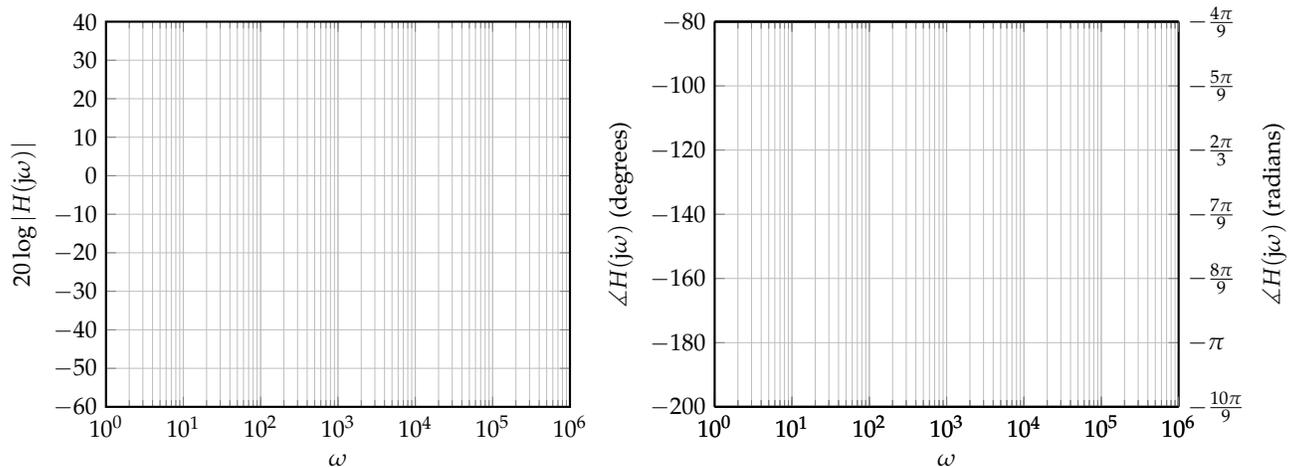
In the previous homework, you analyzed the following circuit in phasor domain:



You (hopefully) determined that $Z_2 = R = 10 \text{ k}\Omega$ and $Z_1 = \frac{1}{j\omega C}$ with $C = 1 \mu\text{F}$ and $R_1 = 1 \text{ k}\Omega$. This gave you the following transfer function:

$$H(j\omega) = -\frac{R}{R_1} \cdot \frac{1}{1 - j\omega CR_1} \quad (1)$$

Draw the magnitude and phase Bode plots (straight-line approximations of the transfer function) of this transfer function. Blank plots are provided here for you to use.



2. Bode Plots 1

A transfer function is given by

$$H(j\omega) = \frac{100}{1 + j\frac{\omega}{1000}} \quad (2)$$

Sketch the asymptotic magnitude and phase Bode plots to scale. What is the value of the half-power frequency?

3. Bode Plots 2

Sketch the asymptotic magnitude and phase Bode plots to scale for the transfer function

$$H(j\omega) = 10 \frac{1 - j\frac{\omega}{100}}{1 + j\frac{\omega}{100}} \quad (3)$$

4. Bandpass Filter: Lowpass and Highpass Cascade

Consider an input signal that is composed of the superposition of:

- $A_p := 20$ mV level pure tone at frequency $f_p := 60$ Hz and phase ϕ_p corresponding to power line noise.
- $A_v := 1$ mV level pure tone at frequency $f_v := 600$ Hz and phase ϕ_v corresponding to a voice signal.
- $A_f := 10$ mV level pure tone at frequency $f_f := 60$ kHz and phase ϕ_f corresponding to fluorescent light control electronics noise.

We would like to keep the 600 Hz tone, which could correspond to a voice signal.

NOTE: The phases ϕ are symbolic – we do not provide numerical values – but the amplitudes A are not symbolic.

- (a) Write the $V_{\text{in}}(t)$ that describes the above input in time domain, in the following format:

$$V_{\text{in}}(t) = A_p \cos(2\pi f_p t + \phi_p) + A_v \cos(2\pi f_v t + \phi_v) + A_f \cos(2\pi f_f t + \phi_f) \quad (4)$$

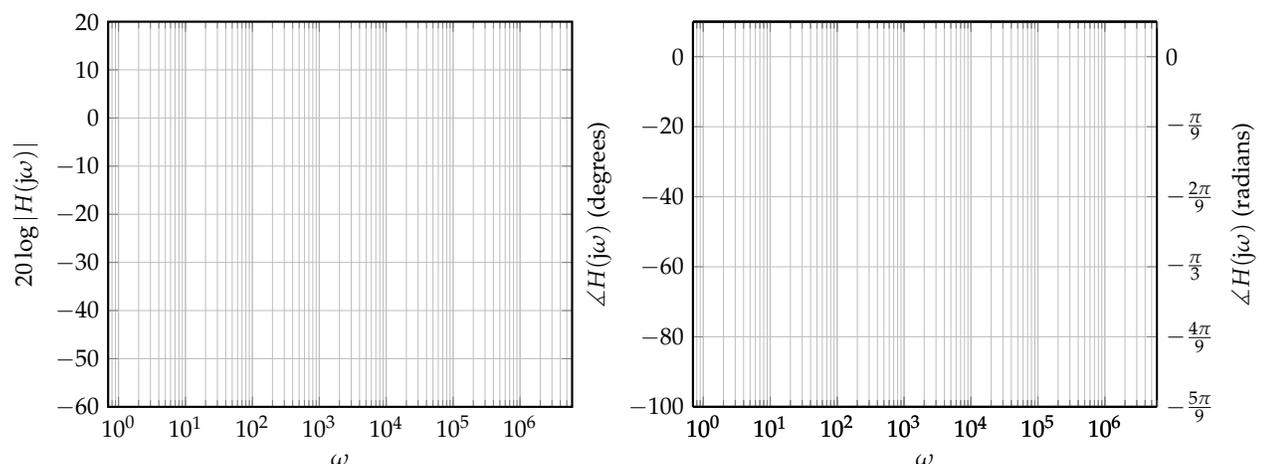
- (b) What are the angular frequencies (i.e., $\omega_p, \omega_v, \omega_f$) involved and the phasors associated with each tone? Remember that the frequencies of the tones are provided in Hz. To convert these frequencies to angular frequencies, we use $\omega = 2\pi f$.

NOTE: This scenario is common in applications; usually, the data collected is in "regular" frequencies, but the analysis requires angular frequencies.

- (c) To achieve your goal of keeping the voice tone but rejecting the noise from the power-lines and fluorescent lights, at what frequency do you want to have the cutoff frequency for the lowpass filters?

(*HINT:* To arrive at a unique solution consider computing the geometric mean (the analogous quantity to the arithmetic mean on a log scale) of the two frequencies of interest.)

- (d) Draw the Bode plots (straight-line approximations of the transfer function) for the magnitude (using $20 \log |H(j\omega)|$) and phase of the lowpass filter.

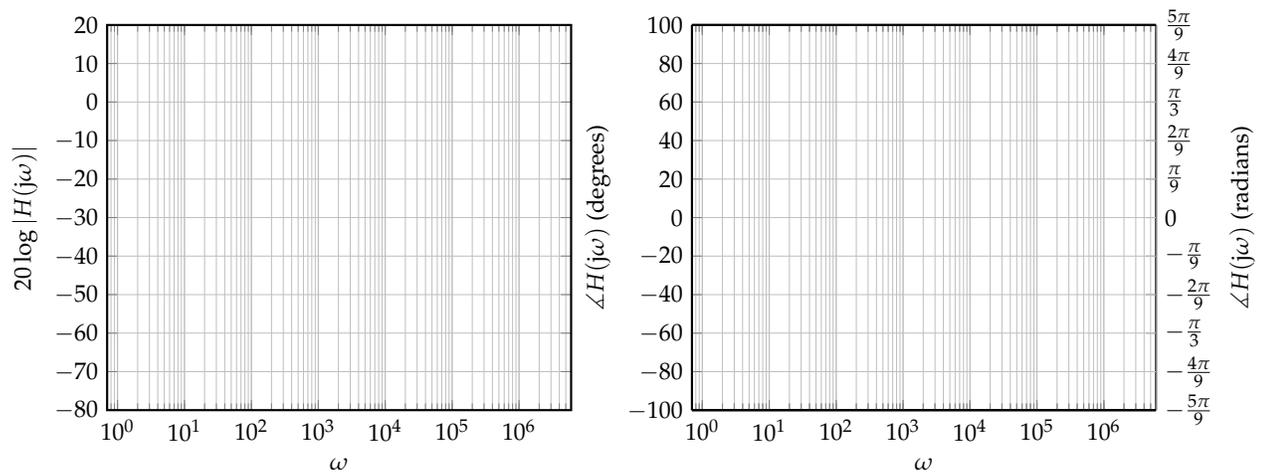


- (e) To achieve your goal of keeping the voice tone but rejecting the noise from the power-lines and fluorescent lights, **at what frequency do you want to have the cutoff frequency for the highpass filters?**

(HINT: To arrive at a unique solution consider computing the geometric mean (the analogous quantity to the arithmetic mean on a log scale) of the two frequencies of interest.)

- (f) **Draw the Bode plot (straight-line approximations to the transfer function) for the magnitude (using $20 \log |H(j\omega)|$) and phase of the highpass filter.**
- (g) For the following questions, assume your cut-off frequencies for lowpass and highpass are 6 kHz and 189 Hz respectively. Suppose that you only had 1 μF capacitors to use. **What resistance values would you choose for your highpass and lowpass filters so that they have the desired cutoff frequencies?**
- (h) The overall bandpass filter that is created by cascading the lowpass and highpass with ideal buffers in between. **Draw the Bode plot (straight-line approximations to the transfer function) for the magnitude and phase of the overall bandpass transfer function.**

(HINT: You should think about how the Bode plot of a cascade of two filters can be derived based on the Bode plots of the lower-level filters.)



- (i) Suppose that the bandpass filter does not have enough suppression at 60 Hz and 60 kHz. You decide to use a cascade of three bandpass filters (with unity-gain buffers in between) (as shown in Figures 1 and 2). **What are the phasors for each of the frequency tones after all three bandpass filters?**

(HINT: Remember how you determined the transfer function of the bandpass filter from the transfer functions of the lowpass and highpass filters.)

Feel free to use a computer to help you evaluate both the magnitudes and the phases here.

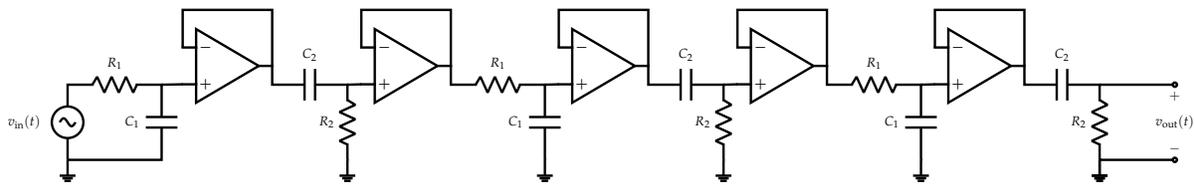


Figure 1: "Time-domain" circuit: Cascade of the three bandpass filters, using buffers to avoid loading.

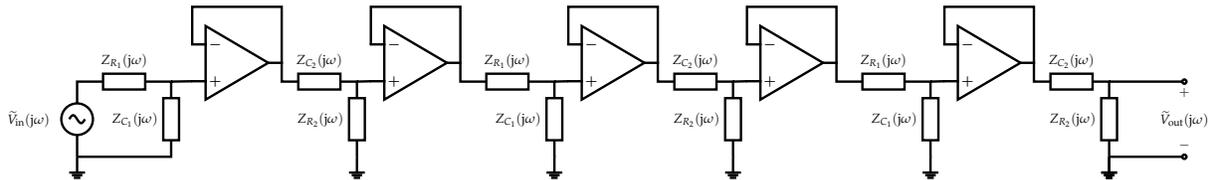
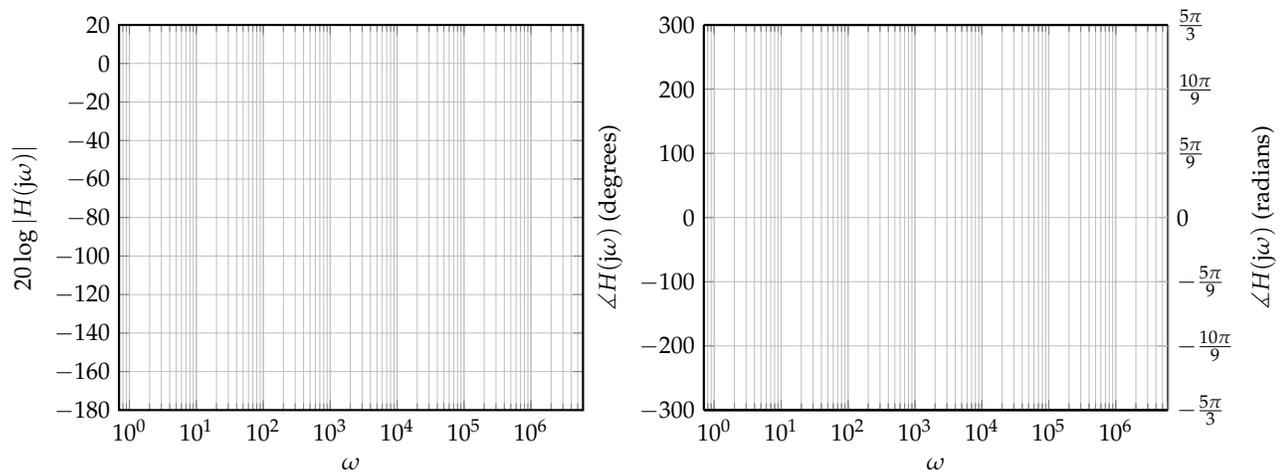


Figure 2: "Phasor-domain" circuit: Cascade of the three bandpass filters, using buffers to avoid loading.

- (j) Draw the Bode plots (straight-line approximations to the transfer function) for the magnitude and phase of the 3rd order bandpass filter. To highlight the difference between the 3rd and 1st order filters, please draw both Bode plots on a single figure.



- (k) Write the final time domain voltage waveform that would be present after the filter.

Contributors:

- Yen-Sheng Ho.
- Sidney Buchbinder.
- Ayan Biswas.
- Druv Pai.
- Antroy Roy Chowdhury.
- Anant Sahai.
- Kris Pister.
- Pavan Bhargava.
- Kourosh Hakhamaneshi.
- Michael Danielczuk.