1 Scalar feedback control

Suppose that x has the following discrete-time dynamics:

$$x(t+1) = \lambda x(t) + bu(t), \quad x(0) = x_0 \tag{1}$$

a) Assuming that $x_0 = 1$ and $u = 0$, sketch $x(t)$ for a few time steps for $\lambda \in \{-1.1, -1, -0.5, 0.5, 1, 1.1\}$.

b) What values of λ will result in convergence of x to its equilibrium? A scalar system having such a λ is called *stable*.

c) If $u(t) = u_0$ and the system is stable, what does x converge to? Sketch stable trajectories of x for $\lambda = 0$, $\lambda < 0$, and $\lambda > 0$.

d) If $x(t+1) = \lambda x(t) + bu(t)$ is unstable, describe feedback laws $u(t) = kx(t)$ that stabilize the equilibrium $x = 0$.

e) Now, consider the continuous time system

$$\frac{d}{dt}x(t) = \lambda x(t) + bu(t) \tag{2}$$

Consider the case where this system is unstable ($\lambda \geq 0$). Design a feedback law $u(t) = kx(t)$ which stabilizes the equilibrium $x = 0$. You can assume that $b > 0$.

2 Eigenvalues Placement in Discrete Time

Consider the following linear discrete time system

$$\vec{x}(t+1) = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \tag{3}$$

a) Is this system controllable?

b) Is the linear discrete time system stable?

c) Derive a state space representation of the resulting closed loop system using state feedback of the form $u(t) = \begin{bmatrix} k_1 & k_2 \end{bmatrix} \vec{x}(t)$

d) Find the appropriate state feedback constants, k_1, k_2 in order the state space representation of the resulting closed loop system to place the eigenvalues at $\lambda_1 = -\frac{1}{2}, \lambda_2 = \frac{1}{2}$

e) Suppose that instead of $\begin{bmatrix}1\\0\end{bmatrix} u(t)$ in (3), we had $\begin{bmatrix}1\\1\end{bmatrix} u(t)$ as the way that the discrete-time control acted on the system. Is this system controllable from $u(t)$?

f) For the part above, suppose we used $[k_1, k_2]$ to try and control the system. What would the eigenvalues be? Can you move all the eigenvalues to where you want? Give an intuitive explanation of what is going on.