Sampling theorem

Let $x$ be continuous signal bandlimited by frequency $\omega_{max}$. We sample $x$ with a period of $T_s$.

Given the discrete samples, we can try reconstructing the original signal $f$ through sinc-interpolation where $\Phi(t) = \text{sinc} \left( \frac{t}{T_s} \right)$

$$\hat{x}(t) = \sum_{n=-\infty}^{\infty} x[n] \Phi(t - nT_s)$$

We define the **sampling frequency** as $\omega_s = \frac{2\pi}{T_s}$. The Sampling Theorem says if $\omega_{max} < \frac{\pi}{T_s}$, or $\omega_s > 2\omega_{max}$, then we are able to recover the original signal, i.e. $x = \hat{x}$.

1 Sampling Theorem basics

Consider the following signal, $x(t)$ defined as,

$$x(t) = \cos(2\pi t). \quad (1)$$

a) Sketch the signal $x(t)$, for $t \in [0, 4]s$.

b) Sketch discrete samples of $x(t)$ is the signal is sampled at a period of

i) $\frac{1}{4}T_s$

ii) $\frac{1}{2}T_s$

iii) $1T_s$

iv) $2T_s$
How would you reconstruct a continuous signal $\hat{f}(t)$ if you only had the discrete samples for reconstruction?

c) What is the maximum frequency, $\omega_{\text{max}}$, in radians per second? In Hertz?
d) If I sample every $T$ seconds, what is the sampling frequency?

e) What is the smallest sampling period $T$ that would result in an imperfect reconstruction?

f) Repeat part (b), for the signal

$$y(t) = \sin(2\pi t)$$
2 Aliasing

Consider the signal $x(t) = \sin(0.2\pi t)$.

a) At what period $T$ should we sample so that sinc interpolation recovers a function that is identically zero?

b) At what period $T$ can we sample at so that sinc interpolation recovers the function $f(t) = -\sin\left(\frac{\pi}{15}t\right)$?