

Discussion 7B

1. System Identification by Means of Least Squares

(a) Consider the scalar discrete-time system

$$x[i + 1] = ax[i] + bu[i] + w[i] \quad (1)$$

Where the scalar state at timestep i is $x[i]$, the input applied at timestep i is $u[i]$ and $w[i]$ represents some (small) external disturbance that also participated at timestep i (which we cannot predict or control, it's a purely random disturbance).

Assume that you have measurements for the states $x[i]$ from $i = 0$ to ℓ and also measurements for the controls $u[i]$ from $i = 0$ to $\ell - 1$. Further assume $\ell \geq 2$.

Show that we can set up a linear system as in eq. (2) to find constants a and b . How do we solve this system?

$$\underbrace{\begin{bmatrix} x[1] \\ x[2] \\ \vdots \\ x[\ell] \end{bmatrix}}_{\vec{s}} \approx \underbrace{\begin{bmatrix} x[0] & u[0] \\ x[1] & u[1] \\ \vdots & \vdots \\ x[\ell-1] & u[\ell-1] \end{bmatrix}}_D \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{\vec{p}} \quad (2)$$

- (b) What if there were now two distinct scalar inputs to a scalar system

$$x[i + 1] = ax[i] + b_1u_1[i] + b_2u_2[i] + w[i] \quad (3)$$

and that we have measurements as before, but now also for both of the control inputs.

Set up a least-squares problem that you can solve to get an estimate of the unknown system parameters a, b_1, b_2 .

- (c) **What could go wrong in the previous case? For what kind of inputs would make least-squares fail to give you the parameters you want?**

- (d) Now consider the two dimensional state case with a single input.

$$\vec{x}[i + 1] = \begin{bmatrix} x_1[i + 1] \\ x_2[i + 1] \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \vec{x}[i] + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u[i] + \vec{w}[i] \quad (4)$$

How can we treat this like two parallel problems to set this up using least-squares to get estimates for the unknown parameters $a_{11}, a_{12}, a_{21}, a_{22}, b_1, b_2$? Write the least squares solution in terms of your known matrices and vectors (including based on the labels you gave to various matrices/vectors in previous parts). *Hint: What work/computation can we reuse across the two problems?*

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