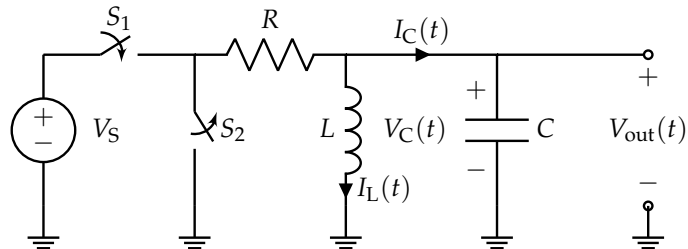


Discussion 6B

1. RLC Circuit with Vector Differential Equations

Consider the following circuit fed by a constant voltage source V_S .



The switch S_1 , open for $t < 0$, closes at $t = 0$, and the switch S_2 , closed for $t < 0$, opens at $t = 0$. Assume $V_C(0) = 0$ and $I_L(0) = 0$.

- (a) **Derive a set of two differential equations, one for $I_L(t)$, the current through the inductor, and one for $V_C(t)$, the voltage across the capacitor.** Write your answer in terms of R , L , C , V_S , and constants.

- (b) **Using your answers from the previous part, create a vector differential equation with the state vector being $\vec{x}(t) = \begin{bmatrix} V_C(t) \\ I_L(t) \end{bmatrix}$.** Write your answers in terms of R , L , C , V_S , and constants.

- (c) Regardless of your answer to the previous part, suppose the vector differential equation is given by

$$\frac{d}{dt} \vec{x}(t) = \underbrace{\begin{bmatrix} -4 & -6 \\ \frac{1}{2} & 0 \end{bmatrix}}_A \vec{x}(t) + \underbrace{\begin{bmatrix} 4 \\ 0 \end{bmatrix}}_{\vec{b}} V_S \quad (1)$$

First, find the eigenvalues of the matrix A .

- (d) **Next, find the eigenvectors that will form your V basis.**

(e) **Now, in order to diagonalize the system, write A in terms of V , V^{-1} , and Λ .**

(f) **With $\vec{x}(0) = \vec{0}$, solve for $\vec{x}(t)$ and find the asymptotic/steady-state behavior as $t \rightarrow \infty$.**