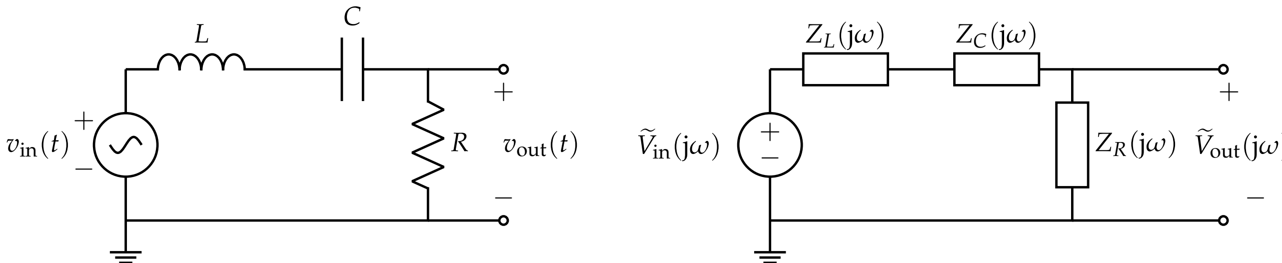
Discussion 5B

1. Band-Pass Filter

It is quite common to need to design a filter which selects only a narrow range of frequencies. One example is in WiFi radios, it is desirable to select only the 2.4GHz frequency containing your data, and reject information from other nearby cellular or bluetooth frequencies. This type of filter is called a band-pass filter; we will explore the design of this type of filter in this problem.



(a) Circuit in "time domain" **(b)** Circuit in "phasor domain"

(a) **Write down the transfer function** $H(\mathrm{j}\omega) = \frac{\widetilde{V}_{\text{out}}(\mathrm{j}\omega)}{\widetilde{V}_{\text{in}}(\mathrm{j}\omega)}$ **for this circuit as a rational function of** $\mathrm{j}\omega$ (make sure the numerator and denominator are both polynomials of $\mathrm{j}\omega$).

(b) Consider the inductor, capacitor, and resistor connected in series. **Write down the impedance of the series RLC combination in the form $Z_{RLC}(j\omega) = A(\omega) + jX(\omega)$, where $A(\omega)$ and $X(\omega)$ are real valued functions of ω. At what frequency ω_0 does $X(\omega_0) = 0$?** (i.e. at what frequency is the impedance of the series combination of RLC purely real — meaning that the imaginary terms coming from the capacitor and inductor completely cancel each other. This is called the *resonant frequency*.) **Describe what the equivalent impedance of the inductor and capacitor is at resonance.**

(c) **Find the ratio of the magnitude of the phasors of inductor voltage to resistor voltage at resonance ($\omega = \omega_0$).** For a series resonance circuit, this is called the **quality factor** (Q) of the circuit and is an important metric to pay attention to (as we will see).

(d) **Show that the transfer function found in part (a) can be written in the form**:

$$H(j\omega) = \frac{1}{1 + jQ(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega})} \tag{1}$$

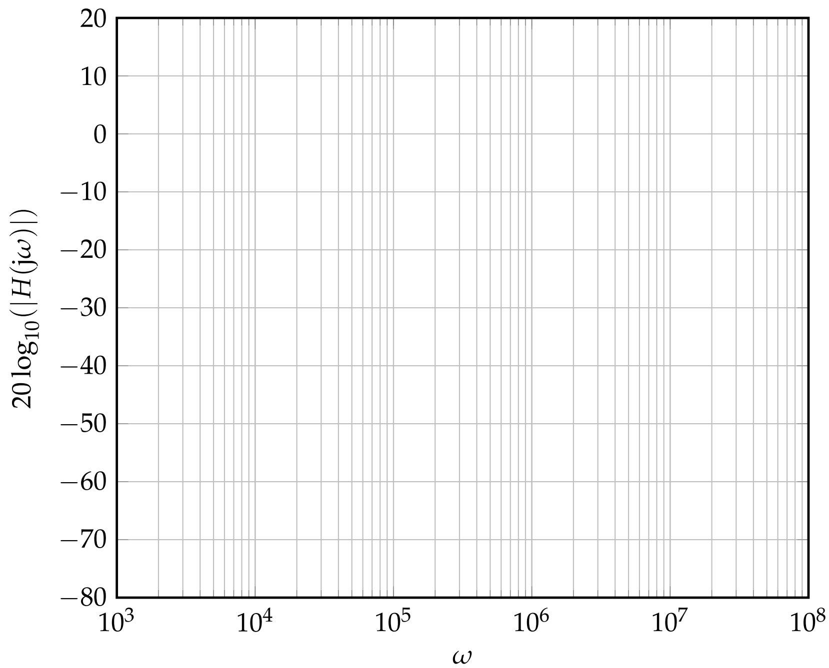
(e) With some derivation, we could show that if we have a high quality factor Q, we could find that the cutoff frequencies will be approximately:

$$\omega_c = \omega_0 \pm \frac{B}{2} \tag{2}$$

where B is the bandwidth and can be calculated as $B = \frac{\omega_0}{Q}$.

Suppose we have $R = 100\,\Omega$, $C = 10\,\text{nF}$, $L = 10\,\text{mH}$. **Find the cutoff frequencies, and draw a Bode plot for $|H(\mathrm{j}\omega)|$ on the provided plot.** *(HINT: Approximate the transfer function for $\omega < \omega_0$ and $\omega > \omega_0$ and combine this with the knowledge that $H(\mathrm{j}\omega_0) = 1$.)*

Plot of $|H(\mathrm{j}\omega)|_{dB}$ (for you to draw).



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