Discussion 1B

For this discussion, Note 1 and Note 2 as well as the corresponding lectures are prequisite materials to review and have available while completing the problems.

1. **Analyzing an RC Circuit with a Sinusoidal Source (Adapted from Hambley Example 4.4)**

   Assume you are given the following circuit, where the capacitor is initially charged so that $v_C(t) = 1V$.

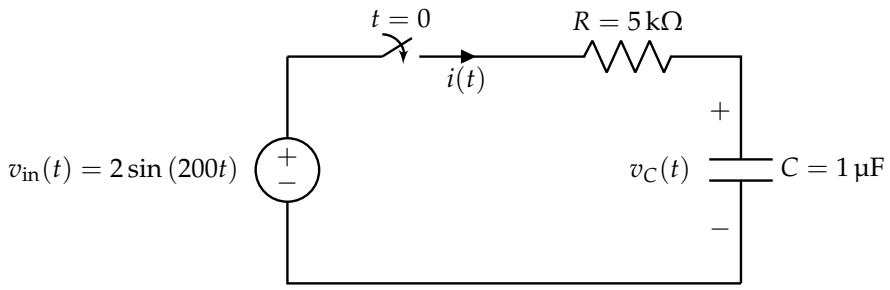


Figure 1

(a) **Set up a differential equation for the voltage $v_C(t)$ across the capacitor in the form:**

$$\frac{dv_C(t)}{dt} = \lambda v_C(t) + u(t) \tag{1}$$

(b) **What is the initial condition of $v_C(t)$? In other words, what is $v(0)$?**

(c) **Solve for the voltage $v_C(t)$ through the capacitor. Also, identify the transient response (homogeneous solution) and the forced response (particular solution) of $v_C(t)$.** You may directly use the fact that the solution to a differential equation in the same form as Equation 1 is:

$$v_C(t) = v(t_0)e^{\lambda(t-t_0)} + e^{\lambda t}\int_{t_0}^{t} e^{-\lambda\theta}u(\theta)\,d\theta \tag{2}$$

(HINT: The following integral might be useful:

$$\int e^{at}\sin(bt) = \frac{1}{b^2+a^2}e^{at}(b\sin(bt) - a\cos(bt)) \tag{3}$$

)

(d) **(OPTIONAL) Solve for the current $i(t)$ through the circuit.**

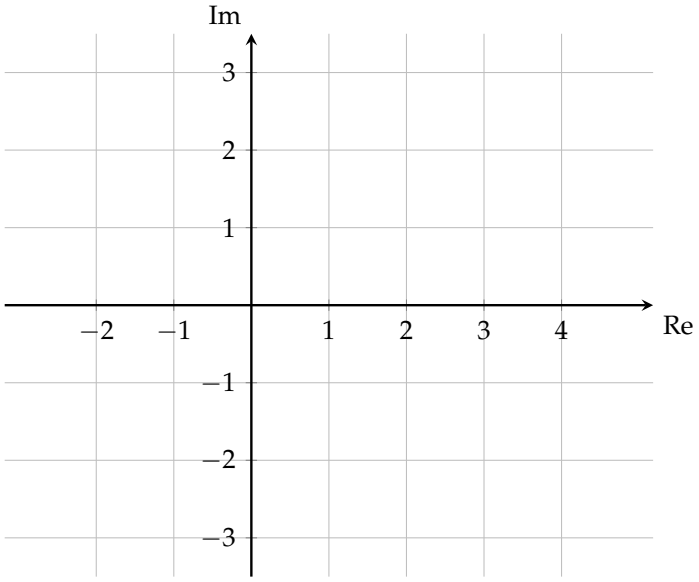
2. Complex Algebra (Review)

(a) **Express the following values in polar forms:** -1**,** j**,** $-\mathrm{j}$**,** $(\mathrm{j})^{\frac{1}{2}}$**, and** $(-\mathrm{j})^{\frac{1}{2}}$**.** Recall $\mathrm{j}^2 = -1$, and the complex conjugate of a complex number is denoted with a bar over the variable. The complex conjugate is defined as follows: for a complex number $z = x + \mathrm{j}y$, the complex conjugate $\overline{z} = x - \mathrm{j}y$.

(b) Represent $\sin(\theta)$ and $\cos(\theta)$ using complex exponentials. (*Hint:* Use Euler's identity $\mathrm{e}^{\mathrm{j}\theta} = \cos(\theta) + \mathrm{j}\sin(\theta)$.)

For the next parts, let $a = 1 - \mathrm{j}\sqrt{3}$ and $b = \sqrt{3} + \mathrm{j}$.

(c) Show the number a in complex plane, marking the distance from origin and angle with real axis.



(d) Show that multiplying a with j is equivalent to rotating the complex number by $\frac{\pi}{2}$ or 90° in the complex plane.

(e) **(Practice)** For complex number $z = x + jy$ show that $|z| = \sqrt{z\overline{z}}$, where $\overline{z}$ is the complex conjugate of z.

(f) **(Practice)** Express a and b in polar form.

(g) **(Practice)** Find ab, $a\overline{b}$, $\frac{a}{b}$, $a + \overline{a}$, $a - \overline{a}$, $\overline{ab}$, $\overline{a}\overline{b}$, and $(b)^{\frac{1}{2}}$.

Contributors:
- Chancharik Mitra.
- Nikhil Jain.
- Neelesh Ramachandran.
- Brian Kilberg.
- Kuan-Yun Lee.
- Maruf Ahmed.
- Anish Muthali.