1 Jacobian Warm-Up

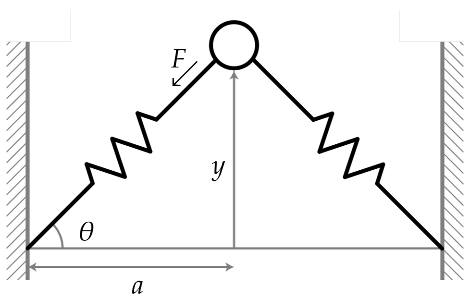
Consider the following function $f : \mathbb{R}^2 \mapsto \mathbb{R}^3$

$$f(x_1, x_2) = \begin{bmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \\ f_3(x_1, x_2) \end{bmatrix} = \begin{bmatrix} x_1^2 - x_2^2 \\ x_1^2 + x_1 x_2^2 \\ x_1 \end{bmatrix}$$

Calculate its Jacobian.

2 Linearization

Consider a mass attached to two springs:



We assume that each spring is linear with spring constant k and resting length X_0. We want to build a state space model that describes how the displacement y of the mass from the spring base evolves. The differential equation modeling this system is $\frac{d^2 y}{dt^2} = -\frac{2k}{m}(y - X_0 \frac{y}{\sqrt{y^2+a^2}})$.

a) Write this model in state space form $\dot{x} = f(x)$.

b) Find the equilibrium of the state-space model. You can assume $X_0 < a$.

c) Linearize your model about the equilibrium.

3 Discretization

Consider a cart of mass M, pushed with a force $u(t)$ with position, $p(t)$, and velocity, $v(t)$. Hence, we have:

$$\frac{\mathrm{d}}{\mathrm{d}t}\, p(t) = v(t)$$
$$\frac{\mathrm{d}}{\mathrm{d}t}\, v(t) = \frac{u(t)}{M}$$

We will apply a constant input between any time $t \in [t, t + T)$. Here T is our time step.

Find a discretized system of equations for this system.