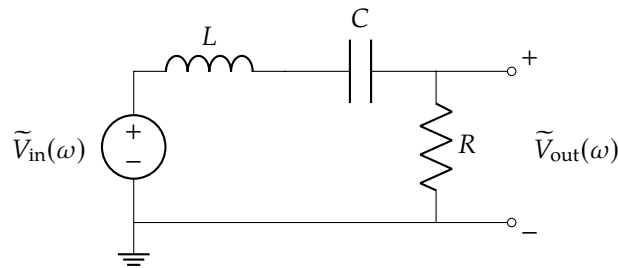


1 Band-pass filter

It is quite common to need to design a filter which selects only a narrow range of frequencies. One example is in WiFi radios, it is desirable to select only the 2.4GHz frequency containing your data, and reject information from other nearby cellular or bluetooth frequencies. This type of filter is called a band-pass filter; we will explore the design of this type of filter in this problem.



- Write down the impedance of the series RLC combination in the form $Z_{RLC}(\omega) = A(\omega) + jX(\omega)$, where $X(\omega)$ is a real valued function of ω .
- Write down the transfer function $H(\omega) = \frac{\tilde{V}_{out}(\omega)}{\tilde{V}_{in}(\omega)}$ for this circuit.
- At what frequency ω_n does $X(\omega_n) = 0$? (i.e. at what frequency is the impedance of the series combination of RLC purely real — meaning that the imaginary terms coming from the capacitor and inductor completely cancel each other.)

What happens to the relative magnitude of the impedances of the capacitor and inductor as ω moves above and below ω_n ? What is the value of the transfer function at this frequency?

- In most filters, we are interested in the cutoff frequency, since that helps define the frequency range over which the filter operates. Remember that this is the frequency at which the magnitude of the transfer function drops by a factor of $\sqrt{2}$ from its maximum value. Notice that the real part of the impedance Z_{RLC} is not changing with frequency and stays at R . What we care about is the frequencies where the imaginary part of the impedance equals $-jR$ to $+jR$.

To do this, we want to see what happens in the neighborhood of ω_n and so express the combined effect of the capacitor and inductor in terms of $\omega = \omega_n + \Delta\omega$, where $\Delta\omega$ is (presumably) a relatively small number compared to ω_n .

Write an expression for $X(\omega_n + \Delta\omega)$, where $\Delta\omega$ is a variable shift from ω_n . Find the values of $\Delta\omega_1$ and $\Delta\omega_2$ which give $X(\omega_n + \Delta\omega_1) = -R$, and $X(\omega_n + \Delta\omega_2) = +R$. Use the approximation that $\frac{1}{1+x} \approx 1 - x$ if $x \ll 1$.

- e) Simplify $X(\omega)$ in two cases, when $\omega \rightarrow \infty$ and when $\omega \rightarrow 0$. Plug this simplified $X(\omega)$ into your previously solved expressions to find the transfer function at high and low frequencies.
- f) A bode plot for a CR filter, a LR filter, and a LCR filter is shown for $L = 9nH$, $R = 0.18\Omega$, and $C = 18.7pF$. Assign each filter to its corresponding line on the plot. Label the locations of the corner frequencies, and describe the behavior of the LCR filter at very high and low frequencies.

