Discussion 15B

1. Using a Nonlinear NMOS Transistor for Amplification

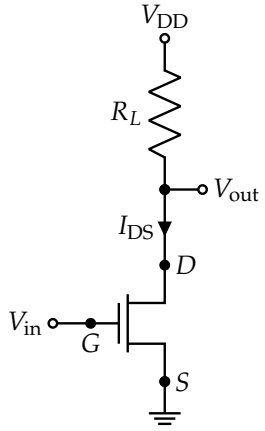
Earlier in the semester, we learned about how transistors can be modeled as a switch, with resistors and capacitors, which allows us to think about how to use transistors for logic circuits. Another important use of transistors is with amplification, and the op-amps we use consist of transistors behaving as amplifiers. This topic is explored more in future circuits classes, but in this discussion, we will start to explore how we can use linearization to model a transistor amplifier circuit.

Consider the following schematic where $V_{DD} = 1.5\,\text{V}$, $R_L = 400\,\Omega$ and the NMOS transistor has threshold voltage $V_{th} = 0.2\,\text{V}$. We are interested in analyzing the response of this circuit to input voltages of the form $V_{in}(t) = V_{in,DC} + v_{in,AC}(t)$, where $V_{in,DC}$ is some constant voltage and $v_{in,AC}(t) = 0.001\cos(\omega t)\text{V}$ is a sinusoidal signal whose magnitude is much smaller than $V_{in,DC}$.

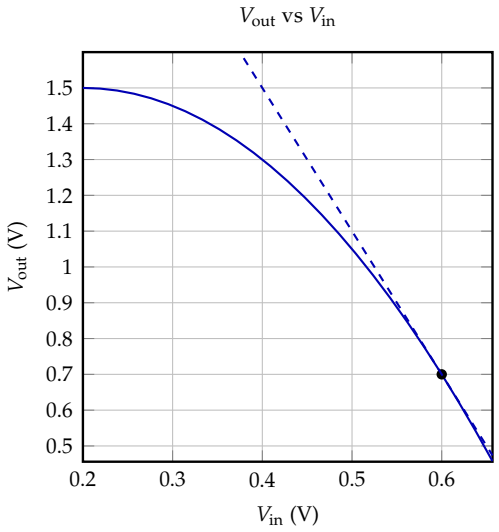
The I-V relationship of an NMOS can be modeled as non-linear functions over different regions of operation. For simplicity, let's just focus on the case when $0 \leq V_{GS} - V_{th} < V_{DS}$ (this is known as the saturation region because the current is approximately constant with respect to V_{DS}, the voltage between the drain and source terminals of the NMOS transistor, but this is out of scope). For this regime of interest, the I-V relationship model we will use is given by

$$I_{DS}(V_{GS}) = \frac{K}{2}(V_{GS} - V_{th})^2 \tag{1}$$

where K is a constant that depends on the NMOS transistor size and properties.



(a) NMOS Transistor circuit



(b) V_{out} vs V_{in} in the regime of interest. Tangent is drawn at the operating point $V_{in,DC} = 0.6\,\text{V}$, $V_{out,DC} = 0.7\,\text{V}$

Figure 1: NMOS figures.

From Ohm's law and KCL, we know that

$$V_{out}(t) = V_{DD} - R_L I_{DS}(t). \tag{2}$$

Note from Figure 1a that $V_{\text{in}} = V_{\text{GS}}$ and $V_{\text{out}} = V_{\text{DS}}$. In Figure 1b, we can see the curve of V_{out} vs V_{in} in the transistor operating regime of interest.

(a) **Using eq. (1) and eq. (2), express $V_{\text{out}}(t)$ as a function of $V_{\text{in}}(t)$ symbolically.** (You can use $V_{\text{DD}}, R_L, V_{\text{in}}, K, V_{\text{th}}$ in your answer.)

Solution:

$$V_{\text{out}}(t) = V_{\text{DD}} - R_L I_{\text{DS}}(t)\Big|_{V_{\text{GS}}=V_{\text{in}}(t)} \tag{3}$$

$$= V_{\text{DD}} - R_L \frac{K}{2}(V_{\text{in}}(t) - V_{\text{th}})^2 \tag{4}$$

A plot of eq. (4) is shown in Figure 1b.

(b) We can decompose the input into constant (i.e., DC) and time-varying (i.e., AC) components to obtain $V_{\text{in}}(t) = V_{\text{in,DC}} + v_{\text{in,AC}}(t)$. **Find $\frac{dV_{\text{out}}}{dV_{\text{in}}}$ at $V_{\text{in}} = V_{\text{in,DC}}$ in terms of R_L and g_m, where $g_m = K(V_{\text{in,DC}} - V_{\text{th}})$ is known as the transconductance gain of the transistor at the provided operating point. Use this to linearize $V_{\text{out}}(t)$ as a function of $V_{\text{in}}(t)$ about $V_{\text{in}} = V_{\text{in,DC}}$.**

Solution: Calculating $\frac{dV_{\text{out}}}{dV_{\text{in}}}$ for $V_{\text{in}} = V_{\text{in,DC}}$ using our answer from the previous part, we find

$$\frac{dV_{\text{out}}}{dV_{\text{in}}} = -R_L K(V_{\text{in,DC}} - V_{\text{th}}) = -R_L g_m \tag{5}$$

To perform linearization around the operating point, we have the following equation:

$$\widehat{V_{\text{out}}}(V_{\text{in}}; V_{\text{in,DC}}) = V_{\text{out}}(V_{\text{in,DC}}) + \frac{dV_{\text{out}}}{dV_{\text{in}}}(V_{\text{in}}(t) - V_{\text{in,DC}}) \tag{6}$$

Using our equations for $V_{\text{out}}(V_{\text{in,DC}})$ and $\frac{dV_{\text{out}}}{dV_{\text{in}}}$, as well as using our definition $v_{\text{in,AC}} = V_{\text{in}}(t) - V_{\text{in,DC}}$ we get the following linear approximation

$$\widehat{V_{\text{out}}}(V_{\text{in}}; V_{\text{in,DC}}) = V_{\text{DD}} - R_L \frac{K}{2}(V_{\text{in,DC}} - V_{\text{th}})^2 - R_L g_m v_{\text{in,AC}}(t) \tag{7}$$

(c) Next, we can also decompose the output V_{out} into DC and AC components to obtain $V_{\text{out}}(t) = V_{\text{out,DC}} + v_{\text{out,AC}}(t)$. **What can we define as $V_{\text{out,DC}}$ from the linearized representation in part 1.b? Assuming that $V_{\text{out}}(t) \approx \widehat{V_{\text{out}}}(t)$ (which occurs when $v_{\text{in,AC}}(t)$ is small), what is the linear approximation for $v_{\text{out,AC}}(t)$?** *(HINT: $V_{\text{out,DC}}$ is a constant value.)*

Solution: The linearized equation is

$$\widehat{V_{\text{out}}}(V_{\text{in}}; V_{\text{in,DC}}) = V_{\text{DD}} - R_L \frac{K}{2}(V_{\text{in,DC}} - V_{\text{th}})^2 - R_L g_m v_{\text{in,AC}}(t) \tag{8}$$

Note that the term $-R_L g_m v_{\text{in,AC}}(t)$ is time-varying due to the $v_{\text{in,AC}}(t)$ term being time-varying. Hence, we may define $V_{\text{out,DC}} := V_{\text{DD}} - R_L \frac{K}{2}(V_{\text{in,DC}} - V_{\text{th}})^2$.

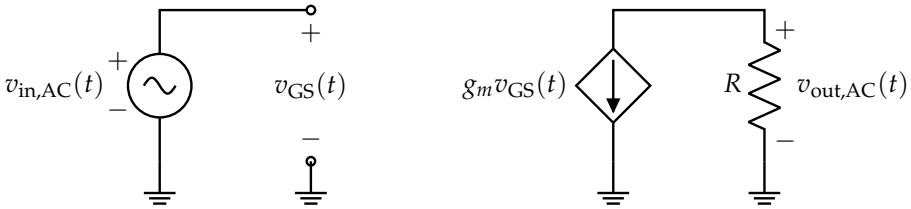
For very small $v_{\text{in,AC}}(t)$, we may write

$$V_{\text{out}}(t) = V_{\text{out,DC}} + v_{\text{out,AC}}(t) \approx \widehat{V_{\text{out}}}(t) = V_{\text{out,DC}} - R_L g_m v_{\text{in,AC}}(t) \tag{9}$$

since the linear approximation is close to the true value of V_{out}. Simplifying this, we obtain

$$v_{\text{out,AC}}(t) \approx -R_L g_m v_{\text{in,AC}}(t) \tag{10}$$

(d) We can create a small signal model for the NMOS transistor amplifier with the following circuit:

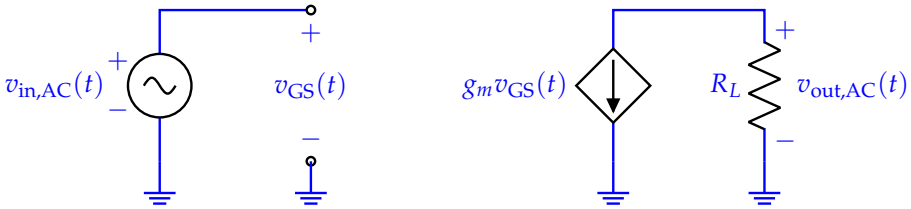


Using the approximation for $v_{\text{out,AC}}(t)$ from the previous part, calculate the resistance value R of the resistor in the small signal model shown above.

Solution: From the circuit provided, we can find the following equation for $v_{\text{out,AC}}(t)$ (note that $v_{\text{GS}}(t) = v_{\text{in,AC}}(t)$):

$$v_{\text{out,AC}}(t) = -(g_m v_{\text{GS}}(t))(R) = -R g_m v_{\text{in,AC}}(t) \tag{11}$$

By looking at our linear approximation $v_{\text{out,AC}}(t) \approx -R_L g_m v_{\text{in,AC}}(t)$, we can identify that $R = R_L$ in this case. Thus, our completed small signal model is the following circuit:



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