

1. Logic Gates

In digital design, we often use ‘synchronous’ circuits, i.e. circuits which evaluate when a clock signal transitions from 0 to V_{DD} . One such implementation, called domino CMOS logic, is shown in Figure 1. Initially $V_{clk} = 0$ (‘reset phase’) for a long time, so the output node is high, i.e. $V_{out} = V_{DD}$ and the capacitor is fully charged, regardless of the values of V_A and V_B . We want to complete the Truth Table 1 during the ‘evaluation phase’.

For cases (ii) and (iv), when V_{clk} transitions from 0 to V_{DD} and V_A and V_B are equal to the values specified in the table, what is V_{out} ? Justify your answer.

Note that if all transistors connected to the output node are switched off, then the capacitor C at the output node ‘holds’ the voltage since there is no charging / discharging path in that case.

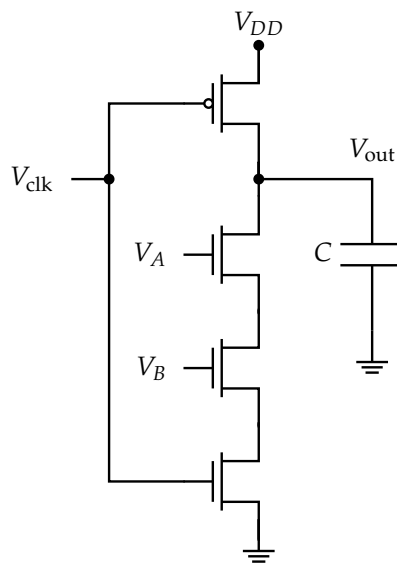


Figure 1: Domino Logic Gate

| Case | V_{clk} | V_A | V_B | V_{out} |
|-------|------------------------|----------|----------|-----------------------------|
| (i) | $0 \rightarrow V_{DD}$ | 0 | 0 | $V_{DD} \rightarrow V_{DD}$ |
| (ii) | $0 \rightarrow V_{DD}$ | 0 | V_{DD} | $V_{DD} \rightarrow ?$ |
| (iii) | $0 \rightarrow V_{DD}$ | V_{DD} | 0 | $V_{DD} \rightarrow V_{DD}$ |
| (iv) | $0 \rightarrow V_{DD}$ | V_{DD} | V_{DD} | $V_{DD} \rightarrow ?$ |

Table 1: Truth Table

2. Analog Signal Processing

In this problem, we will study an example of one of the most common applications in signal processing: removing noise and amplifying the desired signal in a receiver.

In 16B we have learned about filters, so we can selectively remove specific noise frequency bands. Assume that we have a low frequency desired signal $s(t) = \cos(\omega_{\text{sig}}t)$, where $\omega_{\text{sig}} = 10 \frac{\text{rad}}{\text{s}}$, and a high frequency noise $n(t) = 2 \cos(\omega_{\text{noise}}t)$, where $\omega_{\text{noise}} = 1000 \frac{\text{rad}}{\text{s}}$, at the receiver input. We wish to amplify the desired signal and also reject the noise.

- (a) Let's first attempt to use a low-pass filter to achieve this goal. Since we wish to amplify the desired signal, we need to use a low-pass filter with gain > 1 (i.e. use an amplifier combined with a filter). Assume that the op-amps are ideal and follow the golden rules.
- Derive a transfer function for the filter configuration in Figure 2a. Show your work.
 - Derive a transfer function for the filter configuration in Figure 2b. Show your work.
 - Out of the two filter configurations in Figure 2, which one is the low-pass filter? Justify your answer.

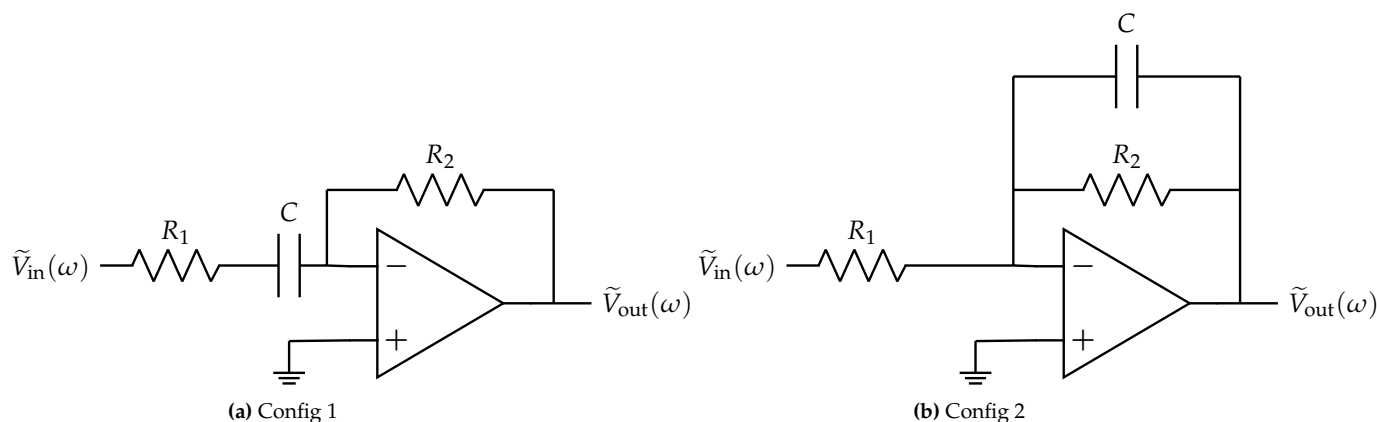
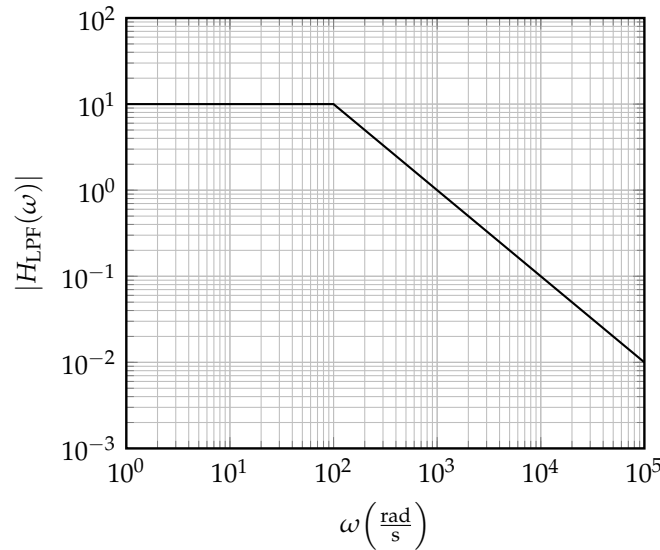


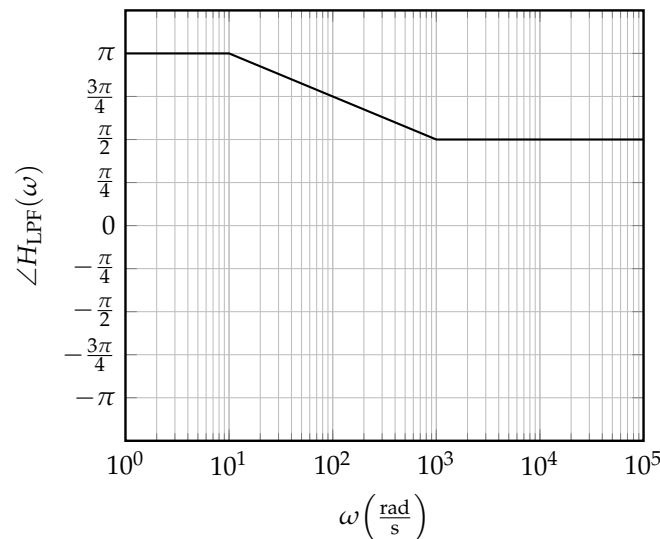
Figure 2: Active filter receiver configurations

(b) Suppose that the transfer function of the low-pass filter with gain from part (a) was $H_{\text{LPF}}(\omega) = -\frac{A}{1+j\frac{\omega}{\omega_c}}$, where the cutoff frequency is $\omega_c = 100\frac{\text{rad}}{\text{s}}$ and the gain is $A = 10$. The Bode plots for the low-pass filter with gain are shown below. Read-off the numerical values corresponding to the appropriate points on the Bode plots.

- What are the magnitude and phase of the filter output signal when the input into the filter is $s(t) = \cos(\omega_{\text{sig}}t)$, where $\omega_{\text{sig}} = 10\frac{\text{rad}}{\text{s}}$? **Derive the time domain expression for the filter output signal.**
- What are the magnitude and phase of the filter output signal when the input into the filter is $n(t) = 2 \cos(\omega_{\text{noise}}t)$, where $\omega_{\text{noise}} = 1000\frac{\text{rad}}{\text{s}}$? **Derive the time domain expression for the filter output signal.**

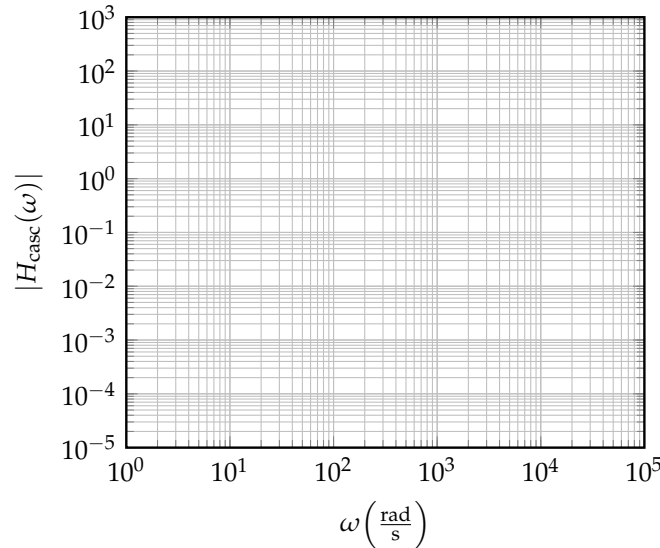
Bode plot of magnitude with $A = 10$ 

Bode plot of phase

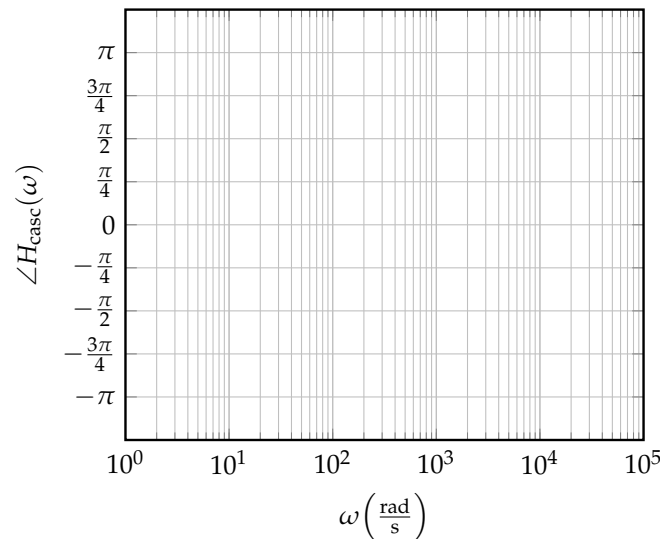


(c) We wish to have the signal be more amplified with respect to the noise. One approach is to cascade two copies of the filter $H_{\text{LPF}}(\omega)$ to make a second-order low-pass filter with gain. Note that it is not necessary to put a unity gain buffer between the two filters, because the V_{out} loading **does not affect** the behavior of this specific filter configuration.

- i. Derive the transfer function $H_{\text{casc}}(\omega)$ of the second-order low-pass filter by cascading 2 of the first order transfer function $H_{\text{LPF}}(\omega) = -\frac{A}{1+j\frac{\omega}{\omega_c}}$ from part (b) with $\omega_c = 100\frac{\text{rad}}{\text{s}}$ and $A = 10$. Show your work.
- ii. Sketch the Bode magnitude and phase plots of $H_{\text{casc}}(\omega)$ on the charts in your answer template.

Bode plot of magnitude with $A = 10$ 

Bode plot of phase



(HINT: Pay attention to the direction of the slopes.)

- (d) Our implementation of the cascaded second-order filter from part (c) uses 2 op-amps. Can we get even more noise attenuation by using a single op-amp? One approach is to use a Notch filter that ideally completely rejects the noise.

Let's consider the cascade of an LC Notch filter with a non-inverting amplifier in Figure 3. We wish to have a notch at the noise frequency so that the noise $n(t) = 2 \cos(\omega_{\text{noise}}t)$, where $\omega_{\text{noise}} = 1000 \frac{\text{rad}}{\text{s}}$, is completely rejected, while the signal $s(t) = \cos(\omega_{\text{sig}}t)$, where $\omega_{\text{sig}} = 10 \frac{\text{rad}}{\text{s}}$, is amplified.

- Derive the transfer function $H_{\text{notch}}(\omega) = \frac{\tilde{V}_{\text{out}}(\omega)}{\tilde{V}_{\text{in}}(\omega)}$ of the filter in Figure 3. Assume that the op-amp is ideal and follows the golden rules. Show your work.
- Using $C = 0.5 \text{ mF}$, find the inductance value L so that the notch (i.e. the frequency at which the magnitude of the transfer function is 0) is at the noise frequency $\omega_{\text{noise}} = 1000 \frac{\text{rad}}{\text{s}}$. Show your work.

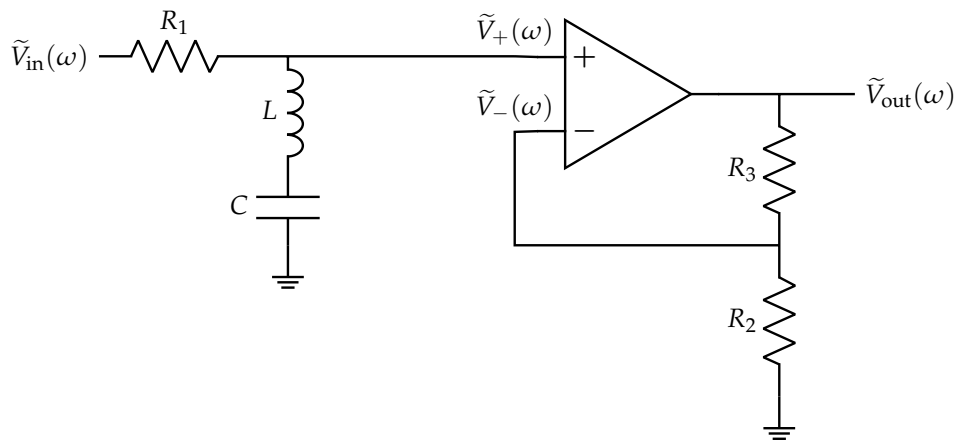
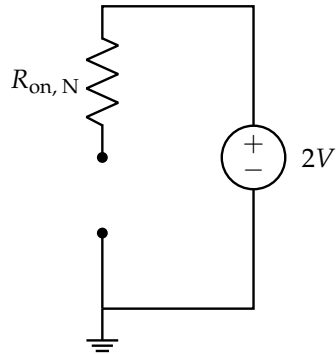
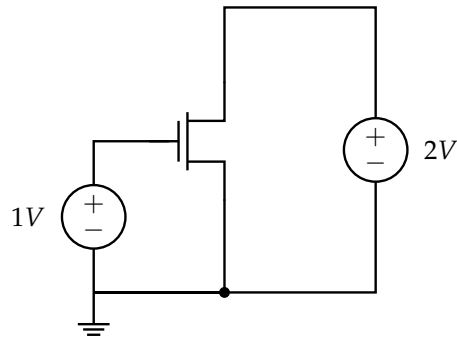


Figure 3: LC Notch filter and non-inverting amplifier

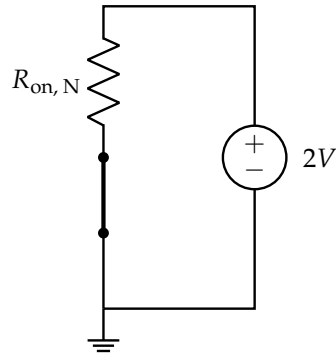
3. Transistor Behavior

For all NMOS devices in this problem, $V_{tn} = 0.5\text{V}$. For all PMOS devices in this problem, $|V_{tp}| = 0.6\text{V}$.

- (a) Which is the equivalent circuit for the right-hand side of the circuit? **Fill in the correct bubble.**



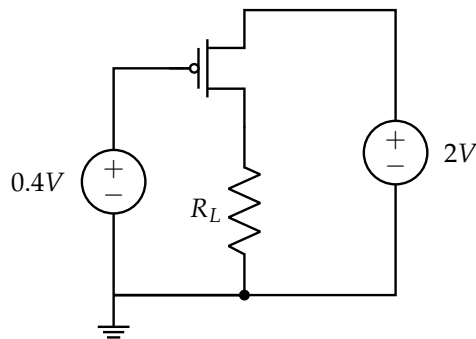
Circuit A

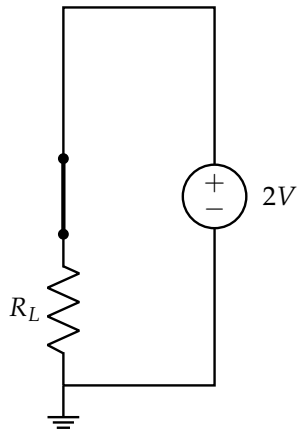


Circuit B

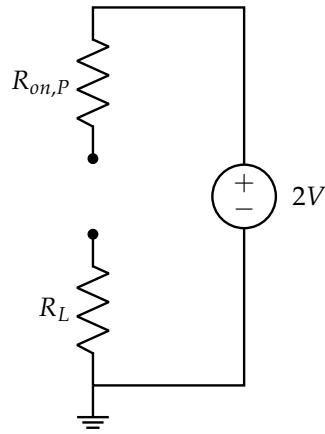
| | A | B |
|---------------------------|-----------------------|-----------------------|
| Equivalent Circuit | <input type="radio"/> | <input type="radio"/> |

(b) Which is the equivalent circuit for the right-hand side of the circuit? **Fill in the correct bubble.**

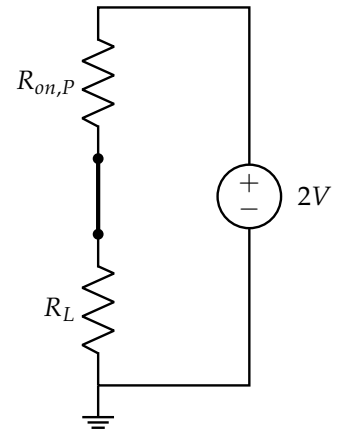




Circuit A



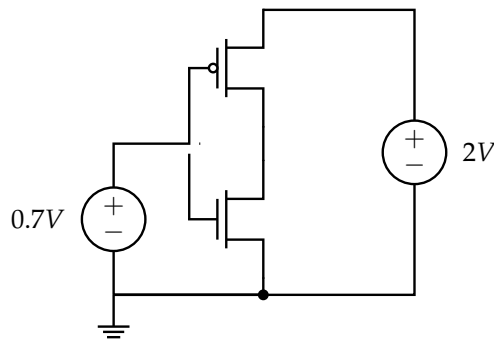
Circuit B

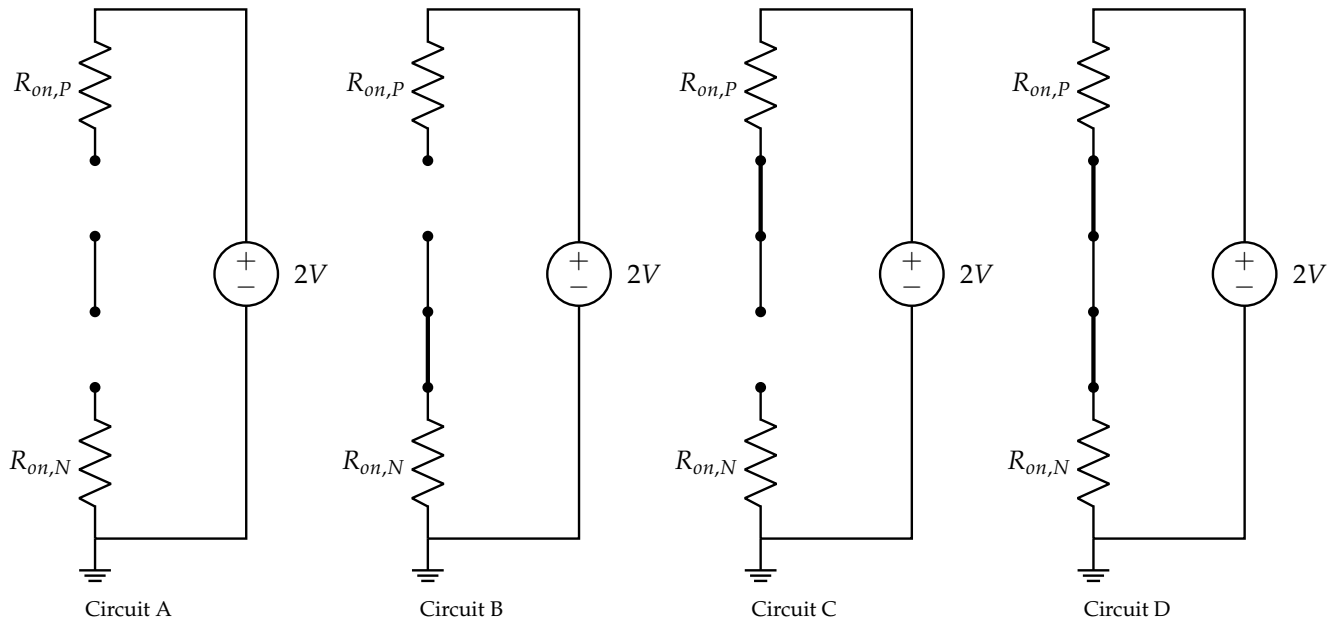


Circuit C

| | A | B | C |
|---------------------------|-----------------------|-----------------------|-----------------------|
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(c) Which is the equivalent circuit for the right-hand side of the circuit? **Fill in the correct bubble.**





| | A | B | C | D |
|---------------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| Equivalent Circuit | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |