Discussion 4B

The following notes are useful for this discussion: Note 6.

1. **Transfer Function Practice**

   Transfer functions take an input phasor and "transform" it into an output phasor. Most of the work we will do with transfer functions is analyzing how it will "respond" to a specific kind of input. We will also design our own transfer functions using common circuit components such as resistors, inductors, and capacitors to achieve some specified behavior. A block diagram of a transfer function is represented in fig. 1. In this discussion, we will learn how to derive $H(\mathrm{j}\omega)$ from a given circuit, and we will analyze how it behaves for certain values of ω.

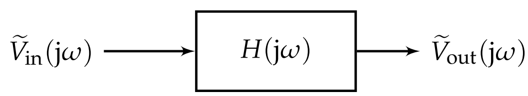
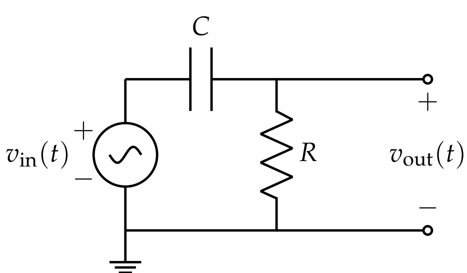


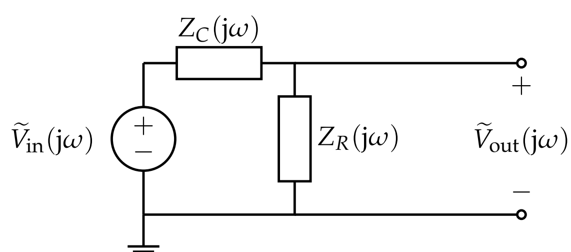
Figure 1: Transfer Function Block Diagram

   Recall that $Z_L = \mathrm{j}\omega L$ and $Z_C = \frac{1}{\mathrm{j}\omega C}$. For large ω, $|Z_L| = \omega L$ becomes large (and becomes small for small ω). On the other hand, for large ω, $|Z_C| = \frac{1}{\omega C}$ becomes small (and becomes large for small ω). In this problem, you'll be deriving some transfer functions on your own. **For each circuit, determine the transfer function** $H(\mathrm{j}\omega) = \frac{\widetilde{V}_{\text{out}}(\mathrm{j}\omega)}{\widetilde{V}_{\text{in}}(\mathrm{j}\omega)}$**. How does** $|H(\mathrm{j}\omega)|$ **respond as** $\omega \to 0$ **(low frequencies), as** $\omega \to \infty$ **(high frequencies)? Is the circuit a high-pass filter, low-pass filter, or band-pass filter?** As practice, sketch a graph (on a log-log plot) of $|H(\mathrm{j}\omega)|$.

   (a) **RC circuit:**

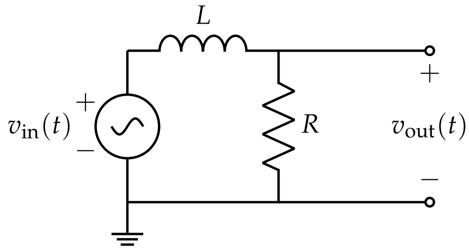


(a) Circuit in "time domain"

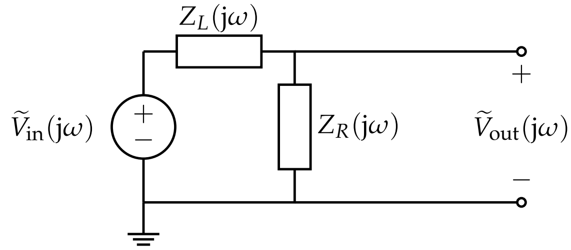


(b) Circuit in "phasor domain"

(b) **LR circuit:**

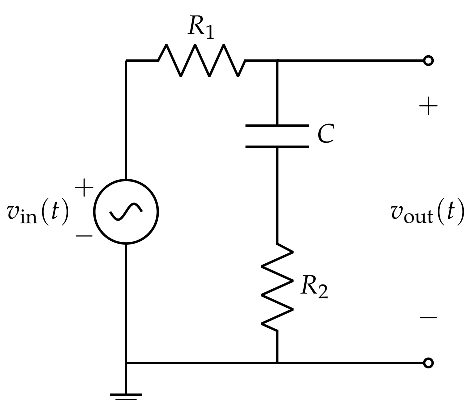


(a) Circuit in "time domain"

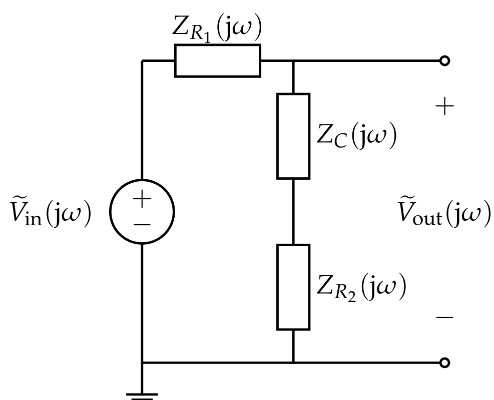


(b) Circuit in "phasor domain"

(c) **(PRACTICE) RCR circuit:**



(a) Circuit in "time domain"



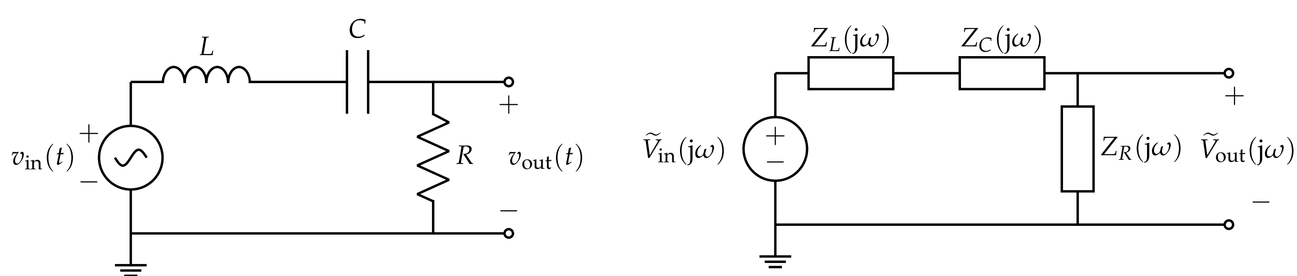
(b) Circuit in "phasor domain"

(d) **Assuming $v_{\text{in}}(t) = 12\sin(\omega_{\text{in}}t)$ compute the $v_{\text{out}}(t)$ using the transfer function computed in part 1.a.** For this part, we assume that $R = 1\,\text{k}\Omega, L = 25\,\mu\text{H}, C = 10\,\mu\text{F}, \omega_{\text{in}} = 100\,\text{rad/s}$. In words, what is the effect of the transfer function in part 1.a on the magnitude of the input signal? Determine whether this behavior is expected given the input signal.

(e) **Visualizing Transfer functions:**
In this part, we visualize the transfer function for different types of circuits in a Jupyter Notebook.

2. Band-Pass Filter

It is quite common to need to design a filter which selects only a narrow range of frequencies. One example is in WiFi radios, it is desirable to select only the 2.4GHz frequency containing your data, and reject information from other nearby cellular or bluetooth frequencies. This type of filter is called a band-pass filter; we will explore the design of this type of filter in this problem.



(a) Circuit in "time domain"

(b) Circuit in "phasor domain"

(a) **Write down the transfer function $H(j\omega) = \frac{\widetilde{V}_{\text{out}}(j\omega)}{\widetilde{V}_{\text{in}}(j\omega)}$ for this circuit.**

(b) Consider the inductor, capacitor, and resistor connected in series. **Write down the impedance of the series RLC combination in the form $Z_{RLC}(j\omega) = A(\omega) + jX(\omega)$, where $A(\omega)$ and $X(\omega)$ are real valued functions of ω. At what frequency ω_n does $X(\omega_n) = 0$?** (i.e. at what frequency is the impedance of the series combination of RLC purely real — meaning that the imaginary terms coming from the capacitor and inductor completely cancel each other. This is called the *natural frequency*.)

(c) **Find an expression for $|H(j\omega)|$. What is $|H(j\omega_n)|$? What is $|H(j\omega_n/10)|$? What is $|H(j10\omega_n)|$?** Rank the three quantities: $|H(j\omega_n)|$, $|H(j\omega_n/10)|$, $|H(j10\omega_n)|$. What do you think the magnitude plot looks like?

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